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The distributional effects of selection and capital accumulation on firm productivity under imperfect capital markets

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Abstract In this evolutionary model, random shocks create differences in the rate of return on capital, while individual saving and investment behavior can reduce these differences over time. Firms with either low total factor productivity (TFP) or a low average return on capital are selected for exit, and new firms enter to take their place. As would be expected, a higher turnover rate improves TFP and reduces its variation. While we show that a higher turnover rate would result in a more positively skewed TFP distribution if exit selection is based directly upon TFP, we find that when we select firms for exit based on their average product of capital, the marginal impact of a higher turnover rate is to more negatively skew the TFP distribution. Overall, our simulations highlight the importance of considering the role selection may play in shaping the distribution of productivity when econometricians seek estimates of firm inefficiency.

Keywords Firm evolution · Selection · Total factor productivity · Productivity distribution

JEL Classification D2 · D9 · O3

1 Introduction

There is growing evidence that productivity improvements result more from a selection process, which reallocates resources from less productive to more productive firms, than from the improvement of the typical firm. A growing number of empirical studies (e.g., Bartelsman and Dhrymes 1998; Dwyer 1998; Foster et al. 1998; Levinsohn and Petrin 1999; and Rigby and Essletzbichler 2000) are finding evidence that aggregate productivity improvements are largely

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explained by the expansion of more productive firms, the contraction of less productive firms, and the entry of new firms. Individually, firms often appear to be trapped in their past choices so that their productivity remains relatively stagnant. Montgomery and Wascher (1988) find evidence that the rate of business failures significantly affects the rate of aggregate productivity growth. As a result of these and other studies, there is a growing recognition of the importance of Schumpeterian creative destruction in any market economy.

If selection processes are important, then the gradual elimination of less productive firms could observably affect the skew of the distribution of firm productivity, not just the mean or variance. That is, it is conceivable that the skew of the productivity distribution could be used to test for the presence of competition in the form of a selection process. To our knowledge, no previous study has recognized this. For example, we might be more likely to observe a more positive skew in a sample of U.S. restaurants where the market is competitive and selection regularly eliminates the poorest performers than in a sample of Japanese banks or Chinese state-owned enterprises subject to government no-failure policies. The success of an economic liberalization effort could conceivably be measured by the observed changes in the skew of the productivity distribution.

However, the issue is not as simple as it may first appear. Productivity must be defined and how a firm is selected for exit is open to question. When we speak of the productivity distribution, we are specifically referring to the distribution of "total factor productivity," or TFP. TFP can be considered a measure of the technology level of the firm or a measure of the extent to which the firm uses the best available business practices. Empirical studies that seek to measure technical inefficiency or use the concept to estimate the location of a stochastic frontier typically rely upon certain assumptions about the TFP distribution.[1]

An alternative measure of productivity is the average product of capital. This productivity measure is more directly associated with the health of the firm in the eyes of the firm's owner, who is forgoing consumption to supply the firm with capital. Below, we illustrate that a selection criterion that forces firms at the lower tail of the TFP distribution to exit will tend to create a positive skew in the TFP distribution. This is not particularly surprising, but provides a basis for comparison. Our primary interest is examining how a selection process that forces out the firms with the lowest average product of capital (which we use as a proxy for firm profitability) will impact the skew of the TFP distribution.

When selection is based upon the productivity of capital, it is not clear how the distribution of total factor productivity will change over time. Suppose two firms with the same capital stock level innovate differently so that the marginal product of capital is higher for the firm that discovers the better technology or business practice. Under normal market conditions, a larger marginal product of capital provides a higher rate of return on capital, which provides a stronger incentive to accumulate capital. However, in the standard model of the firm, the marginal product of capital decreases as capital increases. Thus, the firm that innovates more effectively will find that diminishing returns reduces its competitive advantage. A firm with a better technology could be selected for exit over a firm with a poorer technology if capital accumulation and diminishing returns sufficiently reduces the

[1] Kumbhakar and Orea (2004) and Giannakas et al. (2003) are recent examples of this type of research in this journal.

profitability of the firm. Will selection weed out the worst technologies and business practices, creating a positive skew in the TFP distribution, or can less effective technologies and business practices persist in smaller firms where smaller capital stocks help buttress a higher marginal productivity of capital? Because the answer to this question is not theoretically clear, we use simulation to explore the issue.

In our model, investment must be financed out of current income, so we are implicitly ignoring the important role that financial markets can play in reallocating one firm's savings to another firm's investment. This abstraction allows us to maintain tractability but it is also not exceptionally troublesome. A perfect capital market would provide immediate convergence of capital productivity and rates of return on capital as less innovative firms transferred of their capital to the more innovative firms. So, who would be selected for exit? Our model is useful because the absence of capital market allows the productivity of capital to vary across firms. This allows us to select firms on the basis of capital productivity and makes it possible to produce a model where the productivity of capital for different firms can converge over time.

In our model we also do not allow immediate TFP catch up through imitation or free discrete jumps to the best available technology, for this would obviously generate a negative skew in the TFP distribution. Instead, we assume that firms are locked into their past decisions and investments. New firms can imitate, however, and we assume that they enter with a productivity level that is randomly distributed about the mean productivity level of the existing firms. Existing firms experience productivity shocks each period under the assumption that they try an innovation that may or may not be fruitful. Thus, our model directly recognizes constraints on learning. While these constraints keep the productivity distribution from readily achieving a negative skew, the randomness we assume does not bias the distribution toward a positive skew.

Our primary finding is that if the selection method is based upon average productivity of capital, then a higher rate of selection (entry and exit) leads to a faster-rising mean level of total factor productivity, a slower-rising variance, and a negative marginal effect on skew. The simulation illustrates that large firms with higher total factor productivity may nonetheless be selected for exit because capital accumulation in the face of diminishing returns leads to a relatively low productivity of capital. Firms learn in our model, in the sense that they re-optimize their savings and investment choices each period. However, firms are locked into past investment choices, and negative productivity shocks can suddenly make these choices not only less than optimal, but also non-competitive enough that the firm is selected for exit.

The paper is organized as follows. In Section 2, we present the issues relating to the distribution of firm productivity in some detail, and review some previous work. In Section 3, we present our model of the firm sector. In Section 4, we simulate the model once, examining how the mean, variance, and skew of total factor productivity respond to changes in the selection rate. In Section 5, we simulate the model 6000 times, varying not only the selection rate but also other parameters so as to examine the robustness of impact of changes in the selection rate. Some concluding remarks are presented in Section 6.

2 Optimization, evolution, and firm productivity

Choice plays a different role in traditional neoclassical economic theory than it does in the growing literature on economic evolution. In the neoclassical framework, rational, self-interested, and well-informed individuals with similar preferences and incentives make similar choices, and differences are usually treated as white noise. The assumption that any given activity yields diminishing returns helps keep differences between the best and worst performers to a minimum, and a convergence of performance is typically expected. In contrast, the typical model in evolutionary biology presents individuals as being constrained by processes beyond their individual control, so individual choices are not very important. The possibility of increasing returns makes increasing divergence a possibility, while path-dependence may also amplify initially minor differences.

Rational decision-making may arise in an evolutionary model. However, to do so, it must evolve over time. Textbook biology presents four conditions-variation, selection (competition), transmission (heredity), and iteration (time)-as being necessary for evolution. Pinker (1997) has argued that, if behavior has a genetic component, evolution can result in the selection of unintentional optimizing behavior, given enough time. Likewise, Alchian (1950) demonstrated that rational profit-maximizing behaviors could result from an evolutionary selection process even if firm managers were not particularly rational. Thus, there is reason to assume that prior evolutionary processes have created a population of forward looking, rational decision-makers, but that continuing evolutionary processes prevent decision-makers from actually implementing their plans very far into the future.

We use the optimizing framework of Ramsey (1928), Cass (1965), and Koopmans (1965) to make our agents forward-looking, while introducing an evolutionary process that meets these four conditions and forces the forward-looking agents to repeatedly update their plans.[2] As the economy unfolds iteration by iteration in our experiment, we can observe the impact of the optimizing behavior combined with the evolutionary process on the distribution of firm total factor productivity (TFP).

Changes in TFP can lead to differences in firm profitability. Factors affecting TFP could include production technologies, managerial methods, and intangible input qualities. Let a_{it} denote the TFP of firm i during period t. A typical model of production is

$$y_{it} = a_{it} f(k_{it}, l_{it}) \tag{1}$$

where y_{it}, k_{it}, and l_{it} are the output, capital input, and labor input levels for firm i at time t.

Holding the effects of functional form constant, Fig. 1a presents a TFP distribution which would result if the errors firms make in their technology and management choices are random, but normally distributed around some central tendency.

[2] In doing so, we are following Jovanovic (1982), who cites evidence that small firms grow faster than large firms but are also more likely to fail, and creates a model of "'noisy' selection" to explain why.

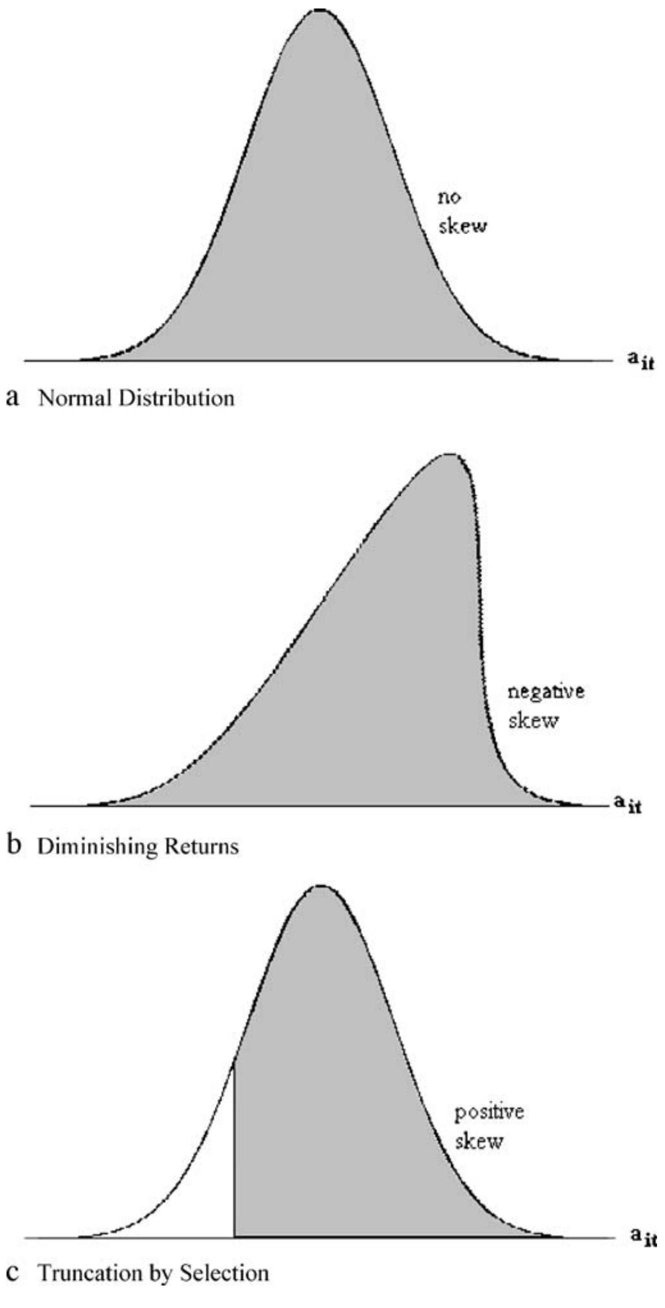

Fig. 1 Productivity distributions under alternative scenarios. **a** normal distribution; **b** diminishing returns; **c** truncation by selection

In contrast to this distribution, it is much more common to assume that TFP is constrained by and gravitates toward some best-practice frontier $\overline{a_t}$ so that $a_{it}\overline{a_t}$. Firms that fail to conform to the best available practices, whether through error or poorly-designed incentives, will produce inside the observable frontier rather than on it. The stochastic frontier econometric approach, developed by Aigner et al. (1977), assumes productivity differentials can be decomposed into two parts, so that

$$a_{it} = \overline{a_t} + u_{it} + \nu_{it}. \tag{2}$$

Here, u_{it} is a normally distributed white-noise measurement error and $\nu_{it} \leq 0$ is the firm's inefficiency, with a half-normal, exponential/gamma, or truncated normal distribution. These distributional assumptions imply observed TFP should possess a significant negative skew as shown in Fig. 1b as firms gravitate toward the best available practices.

However, not all firm-level data sets appear to have the expected negative skew. Green and Mayes (1991) attempted to estimate stochastic frontier production functions for firms in hundreds of different industries across several different countries, and found significant proportions-over half in the United Kingdom and the United States, and over a third in Australia-that either had a positive skew or an excessively small variance in the white-noise component relative to the skewed component. Carree (2002) cites other, similar evidence.

In part because actual data sets often do not have the expected negative skew, the data envelopment approach, developed by Charnes et al. (1978), has become increasingly popular. It uses the equation

$$a_{it} = \overline{a_t} \exp\left(\hat{e}_{it}\right) \tag{3}$$

where $\hat{e}_{it} \leq 0$ is the resulting one-sided firm-specific error term, in order to find the production surface that envelops the observations for the different firms.[3] Although it has the disadvantage of being sensitive to the mismeasurement of firms on the frontier, this non-stochastic approach makes no assumptions regarding the skew of firm TFP.

Why are econometricians finding that different frontier approaches are useful in different cases? We explore the notion that the distribution of TFP may change as the economy evolves. Entrepreneurs are constantly innovating, which leads to new products, new methods of production, and new management methods. Technological progress changes the relative profitability of firms. In this Schumpeterian world of creative destruction, the actions of successful innovators lead to the elimination of those less successful through a competitive selection mechanism like bankruptcy.[4]

If firms make innovations with results that are a priori unknown, then we should expect the results to be essentially random, and the variance of TFP should grow over time. If a selection process then works to remove the firms with the lowest TFP from the population, and new entrants are able to enter with higher TFP than those who have exited, then we might expect that the distribution of TFP would become more positively skewed over time as in Fig. 1c, in contrast to the stochastic frontier approach's assumption of negative skew.

[3] As of June, 2001, when this research began, *Econlit* listed 217 citations for the stochastic frontier approach since 1977, and 365 citations for the data envelopment approach since 1985. In the past few years, citations for the former approach are around half those for the latter approach, suggesting that the former method is gradually being replaced by the latter.

[4] See, for example, Ericson and Pakes (1995) for a description and model of this process.

Because the shape of the TFP distribution is an important issue for econometric work, it is useful to know how selection affects the distribution. Even if the absolute distribution depends a variety of factors, knowing the marginal impact of selection on mean, variance, and skew to may facilitate the construction of a test for the presence of selection. The simulations we perform below inform us about how selection affects the distribution of TFP.

3 The model

Assume a sector of the economy consists of N single-person firms, who must finance their investment out of personal savings. The consumption level c for firm i at time t is chosen by each consumer/firm to maximize inter-temporal utility, with individual-specific discount rate ρ and elasticity of substitution θ. The firm's capital stock k grows by its income less consumption and depreciation, where the depreciation rate is δ. The firm's income is a constant-returns-to-scale Cobb–Douglas function of the firm's TFP level a and its capital stock (since labor for a single person firm is unity). The output elasticity of capital is assumed to be equal across firms, but productivity is assumed to be firm-specific because of continuous innovations in product differences, labor quality, or managerial methods. Following Barro and Sala-I-Martin (1995), the resulting growth model is as follows:

$$\max \int_0^\infty u_i(c_{it}) e^{-\rho_i t} \partial t, \text{ where } u_i(c_{it}) = \frac{c_{it}^{1-\theta_i} - 1}{1-\theta_i} \tag{4}$$

subject to:

$$\dot{k}_i \tfrac{\partial k}{\partial t} = g(c_{it}, k_{it}) = y_{it} - c_{it} - \delta k_{it} \tag{4.a}$$
$$y_{it} = a_{it} f(k_{it}, l_{it}) \tag{4.b}$$
$$f(k_{it}, l_{it}) = k_{it}^{\alpha} 1^{1-\alpha} = a_{it} k_{it}^{\alpha} \tag{4.c}$$
$$c_{it} \leq y_{it} \tag{4.d}$$

The Hamiltonian for this problem is:

$$H(C_{it}, k_{it}, \lambda_{it}) = \frac{c_{it}^{1-\theta} - 1}{1-\theta_i} e^{-\rho_i t} + \lambda^{it} \left(a_{it} k_{it}^{\alpha} - c_{it} - \delta k_{it}\right) \tag{5}$$

with the following first-order conditions:

$$\tfrac{\partial H}{\partial c_{it}} = c_{it}^{-\theta_i} e^{-\rho_i t} - \lambda_{it} = 0 \tag{5.a}$$
$$\tfrac{\partial H}{\partial k_{it}} = \lambda_{it} \left(\alpha a_{it} k_{it}^{\alpha-1} - \delta\right) = -\dot{\lambda}_{it} \tag{5.b}$$
$$\lim_{t\to\infty} \lambda_{it} k_{it} = 0 \tag{5.c}$$

The differential equations obtained from the first order conditions are:

$$\begin{aligned} &(a) \quad \dot{c}_i = \left(\alpha a_i k_i^{\alpha-1} - \delta - \rho_i\right) \tfrac{c_i}{\theta_i} \\ &(b) \quad \dot{k}_i = \left(a_i k_i^{\alpha} - c_i - \delta k_i\right) \end{aligned} \tag{6}$$

In Fig. 2, we show the traditional phase diagram for these two equations. The stable arm for the system shown in the figure is the optimal path of consumption and the capital stock.

3.1 Optimization

Given the current capital stock level k_{i0}, the firm owner's immediate problem is that of finding the optimal current consumption level c_{i0}, which simultaneously determines the optimal current saving and investment levels. Finding this level in the phase diagram is a simple matter, as shown in Fig. 2. However, finding a particular numerical value for current consumption, given values for all of the model's parameters, is more involved. To do so, we use the time-elimination method described by Barro and Sala-I-Martin (1995). It first requires finding the steady-state levels:

$$\begin{aligned} &\text{(a)} \quad \bar{k}_i = \left(\tfrac{\alpha\ a_i}{\rho_i + \delta}\right)^{11-\alpha} \\ &\text{(b)} \quad \bar{c}_i = \left(\alpha\ a_i\ \bar{k}_i^{\alpha} - \delta \bar{k}_i\right) \end{aligned} \tag{7}$$

Then the slope of the stable growth path must be calculated, which is:

$$\frac{\partial c_i}{\partial k_i} = \frac{\dot{c}_i}{\dot{k}_i} = \frac{\left(\alpha\, a_i\, k_i^{\alpha-1} - \delta - \rho_i\right) c_i}{\left(a_i\, k_i^{\alpha} - c_i - \delta\, k_i\right)\theta_i} \tag{8}$$

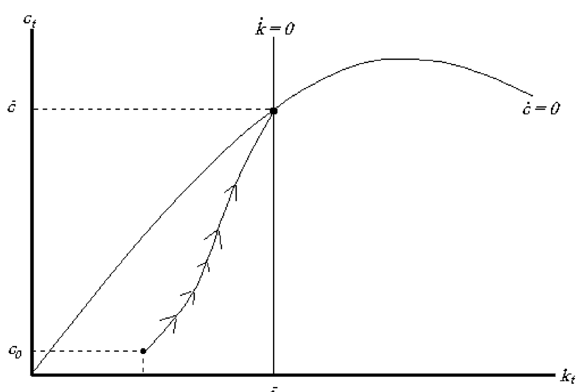

Fig. 2 Phase diagram for Ramsey growth model

This slope must then be evaluated at the steady state. Because the denominator $\dot{k}_i$ and numerator $\dot{c}_i$ are both equal to zero in the steady state, we must evaluate the derivative (Eq. 8) at the steady state using L'Hôpital's rule. After some algebra, we find that at the steady state the derivative (Eq. 8) becomes:

$$\frac{\partial c_i}{\partial k_i} = \frac{\left(\frac{\partial \dot{c}_i}{\partial k_i}\right)}{\left(\frac{\partial \dot{k}_i}{\partial k_i}\right)} = 12\left(\rho_i + \sqrt{\rho_i^2 - 4a_i\,\alpha(\alpha-1)\overline{k}_i^{\alpha-2}\overline{c}_i/\theta_i}\right) \tag{9}$$

The final step is to use Euler's method to follow the stable arm shown in Fig. 2 from the steady state value to an approximation of the location of the point (k_{i0}, c_{i0}). When Z steps (where Z corresponds not to any time period, but instead to the number of steps necessary to accurately trace the nonlinear path) are taken en route from the steady state to the initial state, the change in capital that occurs upon each step is

$$\Delta k_i = \frac{\overline{k}_{it} - k_{i0}}{Z}. \tag{10.a}$$

The path for capital, going backwards in time from the steady state, can then be simulated as

$$k_{i(t-1)} = k_{it} - \Delta k_i, \tag{10.b}$$

where the initial value (working backwards) for k_{it} for is $\overline{k}_i$. We can then simulate the path for consumption using

$$ci_{(t-1)} = c_{ti} - \frac{\partial c_{it}}{\partial k_{it}}\Delta k_i, \tag{10.c}$$

where the initial value for c_{it} for is $\overline{c}_i$, the initial value of $\partial c_{it}/\partial k_{it}$ is given by Eq. (9), and all but the initial value of $\partial c_{it}/\partial k_{it}$ is given by Eq. (8) Upon completing step Z, we have worked our way backwards down the optimal consumption growth path to an estimated location of the point (k_{i0}, c_{i0}). The value obtained for c_{i0} is the estimated optimal current consumption level for the firm that is consistent with the firm's specific capital stock k_{i0}, TFP level a_{i0}, individual parameters ρ_i and θ_i, and shared parameters α and δ.

We assume here that firms must finance their own investments, and this simplifies the optimization process considerably because a market process for capital reallocation is somewhat difficult to simulate with heterogeneous firms.[5] Such grossly imperfect capital markets are an extreme assumption, but our purpose in this model is to create a controlled experiment in which we can isolate the effects of selection on distribution. In a perfectly efficient capital market, we would expect that the variance (and skew) of profit rates would converge to zero in the presence of diminishing returns, since any shock to productivity would immediately be offset by a movement of capital in or out of the firm. We thus choose to consider the other extreme, in which a firm is locked into its own past investments.

[5] Cargill and Parker (2002) used a simulation of a Walrasian tattonment process, but it is computationally demanding.

3.2 Productivity shocks

While each firm faces the same output elasticity α of capital and capital depreciation rate δ, our firm sector is perturbed by varying levels of entrepreneurship, and each firm is constantly experimenting with new innovations. Since the outcomes of these innovations are not known in advance, we follow Jovanovic (1982) by assuming the firm's TFP level is subject to additive random shocks, so that it becomes a random walk: [6]

$$a_{it} = a_{i,t-1} + \varepsilon_{it}, \quad \text{where } \varepsilon_{it} \sim N\left(\mu, \sigma^2\right) \cdot \tag{11}$$

Because variations in the outcomes of innovation efforts affect TFP, while variations in individual preferences affect consumption choices, some individual/firms will be more successful than others even though all are optimizing. In a market in which firms compete for resources, poor performers may exit, either because they choose to do so in order to find more lucrative alternatives or because they are forced to do so because of a mechanism such as bankruptcy and liquidation. We assume firms can be sorted according to observable success criteria, and for simplicity we assume the fraction χ of firms exiting the market in any period is exogenously determined.

In this model, the firm is choosing the optimal rate of savings and investment in each period, in order to maximize intertemporal utility, but these productivity shocks are unexpected. Thus, a negative shock can mean that a firm's past investments suddenly become suboptimal, and a larger firm (which presumably became larger because it was relatively productive in the past) may find its size working against it due to diminishing returns. If the shock is large, then the firm may not survive, but if it does survive then it would reduce its rate of future investment in response. To some extent, this continual re-optimization is a little like learning, though we do not model that explicitly.

3.3 Selection

What success criterion should we use? One reasonable criterion would be to assume that a firm would will voluntarily exit a market if the expected long-run rate of return, calculated using an appropriate risk-adjusted discount rate, is less than that available elsewhere.[7] Cargill and Parker (2002) used such a criterion, shutting down firms when the profit level falls below the wage level less a fixed shutdown cost.

[6] Parker (1995) and Cargill and Parker (2002) assumed that this shock was exponentially normal, of the form $a_{it} = a_{i,t-1}\, e^{\varepsilon_{it}}$. While this has the benefit of ensuring that TFP is always positive, it biases the results towards a positive skew. However, we tested the simulation in Section 4 by re-running it with the exponential shock; in re-estimating the response function, we found the marginal effects not significantly affected.

[7] Of course, bankruptcy as a legal process is rarely so optimal. Managers may be willing to continue operating a loss-making firm if they can hide information on the firm's poor long-run potential from the owners. Creditors with first priority of repayment may force a potentially productive firm to liquidate to ensure that they do not have to accept a loss on their investment.

Here, however, we want the shutdown criterion to be exogenous for later sensitivity tests. To compare a criterion that is not market related to one that is, we consider two alternative selection rules in which some predetermined portion χ of firms in the lower tail of the distribution are shut down:

Rule 1) Following Parker (1995), we rank firms according to TFP level a_{it} and shut them down if their rank is less than or equal to χN.

Rule 2) Firms judge the profitability of their investments by the return on capital. We can write this rate of return as a function of an exogenous output price *P* and an opportunity cost of capital *R*:

$$\frac{\pi_i}{k_i} = P a_{it} k_{it}^{\alpha-1} - R \tag{12}$$

For any *P* and *R*, this rate of return is a monotonic transformation of the average product of capital, and the ordering will be unaffected by the exogenous parameters. We thus rank firms by their average product of capital $a_{it}k_{it}^{\alpha-1}$, and shut them down if their rank is less than or equal to χN. Basing the selection rule on the average product of capital is, of course, insufficient to capture the complexity of this process in the real world, but it is the only available proxy for profitability in this particular model.

4 A simulation

We simulate our model using a fixed population of 1,000 single-owner/manager/worker firms for 20 periods.[8] Relative to reality, the periods are more likely measured in years rather than in months or days, but this could depend upon characteristics of the particular industry. Under our assumption of constant returns to scale, the number of firms is arbitrary. We chose the large number of 1,000 for the number of firms, so that the statistical tests would have considerable power. Because entry and exit can change the characteristics of the industry, allowing the population of firms to change over time might well have an impact. We abstract from this issue here, but recognize that it offers an interesting way to extend this analysis.[9]

Following Barro and Sala-I-Martin (1995), we first choose our non-firm specific diminishing returns and capital depreciation parameter values as α=0.75 and δ=5%. Each of the 1,000 firms in is given randomly-selected preference

[8] We chose 20 iterations because it was long enough to observe changes in the distribution of productivity. We also tested whether our results were sensitive to this duration by rerunning our results for 50 iterations. While coefficient values obviously changed, the sign and statistical significance of these coefficients, as reported in Tables 3, 4, and 5, did not change in either the end or trend regressions.

[9] We thank the editor for raising the issue of changing the number of firms. An interesting extension of this work would be to change the assumptions of the model so that the number of firms is endogenously determined, say by replacing constant returns to scale with decreasing returns. Then, one could systematically examine how changing the economic environment not only changes the number of firms in an economy with selection, but also changes the mean, variance and skew of total factor productivity.

parameters θ and ρ, where θ is distributed uniformly between 0.5 and 3.5, and ρ is distributed uniformly between 0.02 and 0.08.[10] Each firm is endowed with the productivity level a_{i0} =1 and an initial capital stock equal to 10% of the initial steady state:

$$k_{i,0} = 0.10 \times \left(\frac{0.75 \times 1.00}{0.05 + 0.05}\right)^{\frac{1}{1-.75}} \approx 316.4, \text{ for all } i. \tag{13}$$

In each period, firm innovation first leads to productivity shocks as described by Eq. (11). We set the normal distribution parameters as μ=0 and σ=0.04, so there is firm variation but no inherent trend of productivity improvement. Firms produce output given existing productivity and capital stock (Eq. 4c). Firms then each choose their current optimal consumption level (Eqs. 4.a through 10.c), where the number of steps taken in iterating the Euler equation was chosen to be Z=25.[11] Capital stock accumulates through savings (Eq. 4.a) subject to the income constraint (Eq. 4.b), and both for simplicity's sake and because we want current decisions to depend in part on past performance, we assume there is no capital market and firms must rely on their own savings. Finally, firms are sorted on the basis of the selection criterion (either a_i or π/k), and the poorest χ=5% are forced to exit. New entrants begin with the initial capital stock given by Eq. (13) and randomly-selected preference parameters, but their initial TFP level is equal to the output-weighted mean of the surviving firms. We iterate this procedure 20 times.

Figures 3, 4 and 5 show histograms for total factor productivity. [12] Figure 3 shows the results of no selection, or entry and exit, at all. Figures 4 and 5 show the results of implementing exit selection rules 1 and 2, along with replacement entry. A histogram is shown for the initial period t=1 and the final period t=20.

When there is no entry and no exit, as Fig. 3 shows, the distribution of TFP is not skewed in either direction. When we select according to firm's TFP, as Fig. 4 shows, the distribution of TFP acquires a positive skew, as expected. When we select using the average product of capital as our criterion, however, we obtain qualitatively different results. As we can see in Fig. 5, there appears to be no positive skew. This can occur because a firm with a higher level of TFP may nonetheless be selected for exit when it has accumulated a significant amount of capital. The reduction in the average product of capital because of diminishing returns may be more than enough to offset the increase in average product caused by the higher level of TFP.

For each of our three cases we report in Table 1, we give the ending mean values $M(\cdot)$ for firm total factor productivity a_{i}, along with the coefficient of variation and a coefficient of skewness. For some randomly distributed variable x,

[10] The changes in these preference parameters did not significantly influence the results.

[11] We simulated the estimation of current consumption with different parameter sets for many values of Z. We found that consumption estimates were wildly unstable for values of Z below 5, so the stable arm of the growth path is clearly not linear. However, for $Z \geq 10$, we found that the consumption estimates started to converge asymptotically. The estimates obtained for Z=25 were very close to the estimates for T=10,000.

[12] Each case is a separate simulation, and small differences are to be expected due to differences in the random shocks.

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|---------|---------|---------|---------|---------|---------|
0.88     0.92      0.96      1.00      1.04      1.08    1.12
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a Period 1: M(a) = 1.00, V(a) = 0.04, S(a) = -0.12

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|XX      XXOOXXOOXXOOXXOOXXOOXXOOXXOOXXOOXXOOXXOOXXOO    XX
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|---------|---------|---------|---------|---------|---------|
0.48     0.66      0.83      1.00      1.17      1.34    1.51
```

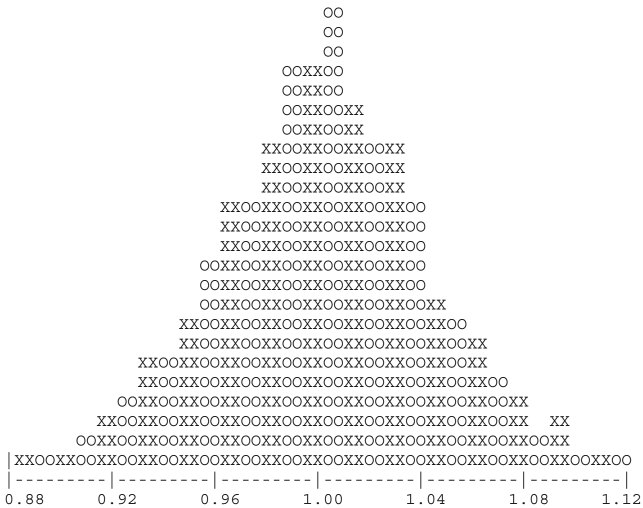
b Period 20: M(a) =1.00, V(a) = 0.17, S(a) = -0.00

Fig. 3 Total factor productivity without selection

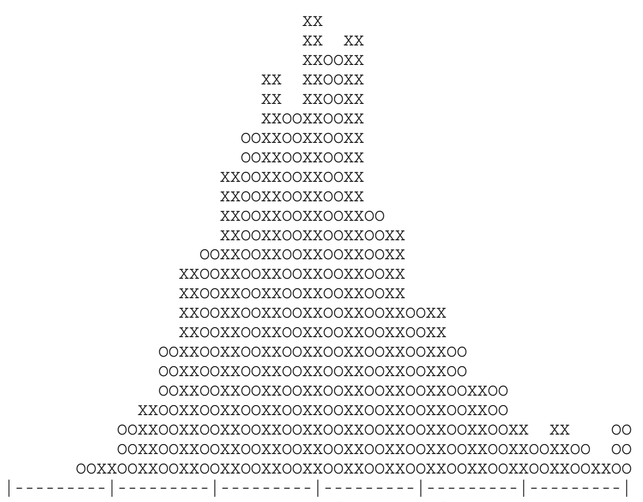
with mean μ_x and standard deviation σ_x, the coefficient of variation is $V(x) = \sigma_x / \mu_x$, and the coefficient of skewness $S(x)$ is

$$S(x) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{x_i - \mu_x}{\sigma_x} \right)^3 . \tag{14}$$

Examining firm productivity, we find that TFP does not improve over 20 periods when there is no selection. Conversely, both selection rules result in



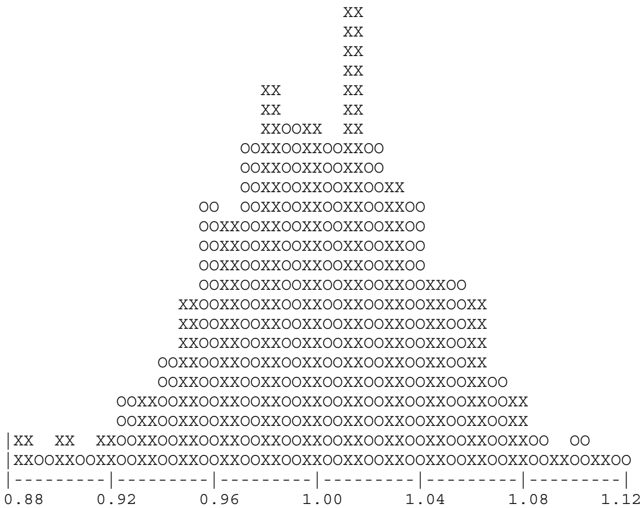
a Period 1: M(a) =1.00, V(a) = 0.04, S(a) = 0.03



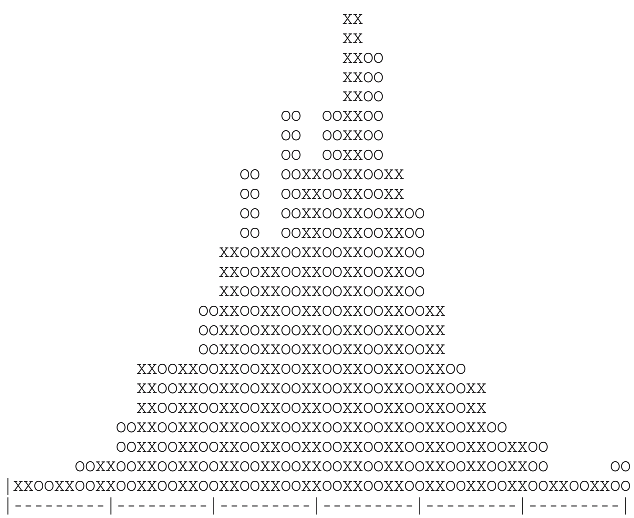
b Period 20: M(a) = 1.16, V(a) = 0.08, S(a) = 0.70

Fig. 4 Total factor productivity with selection based on total factor productivity

significant improvements in total factor productivity. While the variation of TFP still increases over the 20 periods, both selection methods lead to a reduction in the variance relative to having no selection. Consistent with what we can observe in Figs. 4 and 5, the skew of the distribution is very positive under selection rule 1 (using a_{it}), while under selection rule 2 (using $a_{it}k^{\alpha-1}$) the skew is approximately zero, as it is when there is no selection.



a Period 1: M(a) =1.00, V(a) = 0.04, S(a) = -0.02



b Period 20: M(a) = 1.15, V(a) = 0.10, S(a) = -0.02

Fig. 5 Total factor productivity with selection based on capital's average product

Because initial outcomes may differ and because rates of change are not constant, we also examine the results of the simulation by estimating the trend rate of change over the 20 periods. To obtain the rate, we regress each outcome variable on a trend variable *t* using ordinary least squares, where the variable *t* runs from 1 to 20. Because the mean firm output (even initially) is sensitive to parameter assumptions, we normalize the mean of output prior to this regression so that rates of change are comparable, by dividing mean output in each period by its mean value in the first period. For each of the cases, we report the trend coefficients in Table 2.

Table 1 Mean ending values for three cases

	No Selection	Selection Rule 1	Selection Rule 2_
Total factor productivity (a):			
Mean	1.00	1.16	1.15
Variance	0.17	0.08	0.10
Skew	0.00	0.70	−0.02

For the most part, the trend results in Table 2 mirror the end value results in Table 1. Selection results in a faster rising mean of TFP. The coefficient of variation for TFP rises over the 20 periods in all cases, but much more slowly with selection than without. The skew coefficient for TFP rises slightly over time without selection, and rises rapidly under the first selection rule, but it declines under the second selection rule.

5 Estimating the response function

Are the simulation results reported in the previous section robust, or are they idiosyncratic results from our particular parameter estimates? To answer this question, we ran 6,000 separate experiments, with 3,000 experiments for each of the two selection rules. The values of four of our model's exogenous parameters were randomly chosen from uniform distributions for each particular experiment, and then held constant over the 20 iterations of the experiment. Specifically, the output elasticity of capital α was chosen from the 20 to 80% interval, the depreciation rate δ was chosen from the 5 to 15% interval, the standard deviation σ of the TFP shock was chosen from the 1 to 5% interval, and the selection (exit/entry) rate χ was chosen from the 0 to 10% interval.

Using the results obtained for each experiment, we then gather experiment-specific results. For TFP, we measure the mean, the coefficient of variation, and the coefficient of skewness. For all 6,000 experiments, we then include two measures for each: first, we report the values in period 20, the end period; second, we estimate and report trend coefficients to look for consistent changes over time.

We are interested in how the evolution of the firm's TFP is affected by the values of the parameters α, δ, σ, and χ. Therefore, we regress the experimental results on these four parameters, using a separate regression for each selection rule. We consider two alternative specifications-one linear with no interaction and one quadratic with interaction. To aid interpretation of the coefficient estimates, we

Table 2 Trend regression results for three cases

	No Selection	Selection Rule 1	Selection Rule 2
Total factor productivity (a):			
Mean trend	0.0028	0.0096	0.0101
Variance trend	0.0065	0.0017	0.0026
Skew trend	0.0047	0.0289	−0.0102

normalize the data prior to estimation by dividing each data point by the absolute value of its sample mean, so that the coefficient is equal to the elasticity at the mean of the data. We report only the estimated coefficients, not standard errors, but note where the *t*-statistics exceed the 5% (*) or 1% (**) level of statistic significance. To aid in interpreting the regression results reported in Tables 3, 4 and 5, we also use the (†) symbol to denote estimated coefficients (other than the constant) which are statistically significant and consistent in end and trend value for both selection rules, and the (!!) symbol to denote coefficients that are statistically significant and consistent in end and trend values, but reversed in sign between the two selection rules. We are interested in the inconsistencies between selection rules, and particularly in the effects of selection rule 2 because it is a better proxy for profitability.

Table 3 presents the effects of the four exogenous variables α, δ, σ, and χ on the mean level of firm total factor productivity. In the linear regressions, we find that

Table 3 Simulation results for mean of total factor productivity

	Selection rule 1		Selection rule 2		
	End	Trend	End	Trend	
Mean *M(a)*	1.110	0.007	1.073	0.005	
Linear:					
C (constant)	0.831**	−0.920**	0.843**	−0.845**	
α	0.004**	0.113**	0.023**	0.377**	†
δ	0.001	0.012	0.002	0.017	
σ	0.103**	1.219**	0.082**	1.063**	†
χ	0.076**	0.710**	0.031**	0.192**	†
R^2	0.91	0.94	0.79	0.81	
Quadratic:					
C	0.909**	−0.044	0.953**	0.493**	
α	−0.004	−0.084**	−0.068**	−0.975**	†
δ	−0.007*	−0.087**	−0.003	−0.025	
σ	0.013**	0.190**	−0.024**	−0.275**	!!
χ	0.037**	0.367**	0.028**	0.436**	†
α	−0.001	−0.002	0.019**	0.281**	
$\alpha\delta$	0.002*	0.046**	−0.004*	−0.032	
$\alpha\sigma$	0.004**	0.156**	0.028**	0.491**	†
$\alpha\chi$	0.002**	−0.012*	0.029**	0.339**	
δ	0.001	0.006	0.005*	0.042	
$\delta\sigma$	0.002*	0.032**	0.002	0.024	
$\delta\chi$	0.001*	0.008	−0.005**	−0.057**	
σ	0.003**	0.086**	0.014**	0.232**	†
$\sigma\chi$	0.078**	0.661**	0.048**	0.358**	†
$\chi\chi$	−0.021**	−0.154**	−0.035**	−0.442**	†
R^2	0.99	0.99	0.95	0.96	

**The *t*-statistic exceeds the 1% (two-tailed) level of statistical significance
*The *t*-statistic exceeds the 5% (two-tailed) level of statistical significance
†Consistent and significant for both selection methods, in both end and trend
!!Consistent and significant in both end and trend, but sign reverses with selection method

greater variation in the productivity shocks σ and higher rates of selection χ lead to faster TFP growth under either selection rule. Changes in the depreciation rate δ have little effect on TFP under either selection rule. Under selection rule 2, increasing the output elasticity for capital α, which is the same as reducing the amount of diminishing returns, increases TFP; this is because firms with higher TFP and larger capital stocks are less likely to be selected for exit because diminishing returns does not so quickly reduce the productivity of the large capital stock. In the quadratic regressions, we find that the marginal effect of selection is falling at higher selection rates, and the interaction between variation and selection is positive. We also find one of the coefficients changes sign between the two selection methods, but interpretation is difficult in the quadratic case.

Table 4 presents the effects of the four exogenous variables α, δ, σ, and χ on the coefficient of variation for TFP. Greater variation in the productivity shocks leads to greater variation in TFP, of course. However, this effect diminishes since it is negative in its square, and it also declines with greater rates of selection. Greater

Table 4 Simulation results for variance of total factor productivity

	Selection rule 1		Selection rule 2		
	End	Trend	End	Trend	
Mean $V(a)$	0.071	0.002	0.085	0.003	
Linear:					
C	0.516**	1.025**	0.518**	0.966**	
α	−0.008	−0.014	−0.069**	−0.155**	
δ	0.007	0.016	−0.014	−0.020	
σ	0.841**	0.665**	1.000**	1.029**	†
χ	−0.435**	−0.864**	−0.330**	−0.567**	†
R^2	0.89	0.78	0.93	0.83	
Quadratic:					
C	0.149**	0.276**	−0.045	−0.191*	
α	0.001	0.020	0.158**	0.414**	
δ	0.042	0.119	0.013	0.099	
σ	1.453**	1.896**	1.727**	2.453**	†
χ	−0.520**	−1.064**	−0.209**	−0.388**	†
$\alpha\alpha$	−0.002	−0.006	−0.065**	−0.177**	
$\alpha\delta$	−0.000	−0.018	0.017	0.044	
$\alpha\sigma$	0.009	0.026	−0.090**	−0.206**	
$\alpha\chi$	−0.009	−0.022	−0.024**	−0.052**	
$\delta\delta$	−0.012	−0.031	−0.027	−0.094*	
$\delta\sigma$	0.000	−0.004	−0.028*	−0.057*	
$\delta\chi$	−0.011	−0.020	0.044**	0.093**	
$\sigma\sigma$	−0.075**	−0.156**	−0.129**	−0.264**	†
$\sigma\chi$	−0.465**	−0.929**	−0.351**	−0.633**	†
$\chi\chi$	0.282**	0.579**	0.106**	0.208**	†
R^2	0.98	0.96	0.97	0.91	

**The *t*-statistic exceeds the 1% (two-tailed) level of statistical significance
*The *t*-statistic exceeds the 5% (two-tailed) level of statistical significance
†Consistent and significant for both selection methods, both end and trend

selection reduces the variance, but the negative effect declines as the rate of selection rises and it rises with greater variation. A reduction in the extent of diminishing returns reduces the variation of TFP under selection rule 2, a result that does not yet have an intuitively obvious explanation.

Table 5 presents the effects of the four exogenous variables α, δ, σ, and χ on the skew of TFP, and we find that the results are very dependent on the selection rule. A greater selection rate leads to a more negative skew under rule 2, but a more positive skew under rule 1. Greater variation in productivity shocks leads to a more positive skew under selection rule 2, but a more negative skew under rule 1. Reducing the extent of diminishing returns leads to a more positive skew under rule 2, but more negative skew under rule 1. Put differently, for rule 2, which is of more interest, a move from a more negative to more positive skew can result from a lower selection rate, greater variation in the productivity shock, or a reduction in the extent of diminishing returns.

Table 5 Simulation Results for Skew of Total Factor Productivity

	Selection rule 1		Selection rule 2		
	End	Trend	End	Trend	
Mean $S(a)$	0.590	0.028	0.099	−0.002	
Linear:					
C	1.297**	1.547**	0.151**	−0.029	
α	−0.098**	−0.144**	0.185**	0.221**	!!
δ	−0.036	−0.041	0.006	0.026	
σ	−0.111**	−0.146**	0.161**	0.241**	!!
χ	0.501**	0.483**	−0.245**	−0.611**	!!
R^2	0.40	0.27	0.18	0.28	
Quadratic:					
C	0.618**	0.745**	0.786**	1.075**	
α	0.116	0.161	−0.783**	−1.060**	
δ	−0.080	0.148	0.185	0.300	
σ	0.082	−0.139	0.055	0.148	
χ	1.943**	2.117**	−1.044**	−2.666**	!!
$\alpha\alpha$	−0.027	−0.094	0.495**	0.712**	
$\alpha\delta$	−0.042	−0.065	−0.161**	−0.215*	
$\alpha\sigma$	−0.059	0.007	0.063	0.066	
$\alpha\chi$	−0.049	−0.050	0.061*	−0.025	
$\delta\delta$	0.065	−0.042	0.043	0.070	
$\delta\sigma$	−0.032	−0.018	0.023	0.000	
$\delta\delta$	−0.012	−0.020	−0.105**	−0.139**	
$\sigma\sigma$	−0.049	−0.003	0.009	−0.013	
$\sigma\chi$	−0.012	0.002	−0.002	0.046	
$\chi\chi$	−0.681**	−0.778**	0.419**	1.080**	!!
R^2	0.60	0.45	0.30	0.50	

**The *t*-statistic exceeds the 1% (two-tailed) level of statistical significance
*The *t*-statistic exceeds the 5% (two-tailed) level of statistical significance
!!Consistent and significant for both end and trend, but sign reverses with selection method

6 Conclusion

Our model and simulations have allowed us to confirm some intuitively expected impacts of selection and explore impacts on skew of the firm total factor productivity distribution that are less intuitive. Confirming the intuitive, we find that more variable factor productivity shocks and an increased selection rate improve TFP on average, and more variable productivity shocks increase the variance of TFP while greater selection rates reduce the variance of TFP. Exploring the less intuitive impact on skew, we find the effect of selection on the skew depends upon the selection rule. When the selection criterion depends upon the firm's rate of return, the firms with the lowest TFP levels are not necessarily those that are shut down. Consequently, the distribution of TFP need not become more positively skewed as the selection rate rises.

As they stand, our results offer an explanation for why econometricians sometimes find that firm total factor productivity distributions do not match theoretical expectations. It may be that, as in our simulation, the mean, variance, and skew of real world total factor productivity distributions are correlated with the criteria that are selecting firms for exit. A positively skewed distribution of total factor productivity results when the selection criterion places more weight on total factor productivity compared to other influences, while these effects are not found when the selection criterion gives weight to the return on capital, at least when diminishing returns to capital allows smaller firms with lower total factor productivity to catch up. Our simulations suggest econometricians should be cautious about having an expectation for the skew of total factor productivity distribution, and should consider the different roles selection and capital accumulation (or other factors) may play in altering skew.

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