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**Paradoxes of Traffic Flow and Economics of Congestion Pricing**

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**Abstract**

This paper utilizes a unique county-level dataset to examine technical efficiency and technology gap in China's agriculture. We classify the counties into four regions with distinctive levels of economic development, and hence production technologies. A meta-frontier analysis is applied to the counties. We find that although the eastern counties have the highest efficiency scores with respect to the regional frontier but the northeastern region leads in terms of agricultural production technology nationwide. Meanwhile, the mean efficiency of the northeastern counties is particularly low, suggesting technology and knowledge diffusion within region might help to improve production efficiency and thus output.

**JEL Classification:** D24, N55, O13

**Keywords:** China's grain production; County-level; Metafrontier; Stochastic production frontier; Technical efficiency

# **Paradoxes of Traffic Flow and Economics of Congestion Pricing**

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# **Paradoxes of Traffic Flow and Economics of Congestion Pricing**

## **Abstract**

With rapid urbanization and fast growth of income, China sees worsening traffic congestion in all major cities. Previous studies on traffic congestion have emphasized on the supply-side instruments, such as expanding road capacity and improving traffic management. However, transportation researchers have identified three paradoxes in which the usual remedy for congestion – expanding the road system – is ineffective or even counterproductive. This paper has two purposes. One is to present three paradoxes of traffic flow in their general forms. The other is to give the economics of congestion pricing. For the latter, the paper will first address traffic externalities, then propose pricing options, and finally discuss applications. Unlike most early studies, this paper emphasizes on the demand-side policies by examining the behavior of commuters and using pricing mechanism.

Key words: Congestion pricing; externality; traffic paradoxes  
JEL Code: R41, H40

## **1. Introduction**

Since 1990, China has been experiencing rapid urbanization and fast income growth, with its urban population increasing from 301.95 million in 1990 to 542.83 million in 2004 and real per capita GDP increasing from 5233 RMB to 10561 RMB, respectively (NBSC, 2005). One consequence is a dramatic growth of private vehicles. In 1990, China had 5,502,648 private vehicles. This number became 31,596,629 in 2005, 5.47 fold increasing in 15 years. The growth of private vehicles in major cities is even faster. For example, Beijing and Shanghai increased 6.75 and 6.44 fold, respectively during the same period. No question, China sees severe traffic congestion on its urban roads, which not only wastes tremendous amount of time of urban commuters but also causes many more fatal traffic accidents. Because of traffic accidents, China lost 48,271 and 98,738 lives in 1990 and 2005, respectively (SDPC, 2001-2007).

Like other countries, China mostly depends on supply-side policies to mitigate urban congestion, such as through expanding network capacity and improving traffic management. For instance, Beijing has restricted odd and even plate number vehicles for inner city on even and odd dates since June 17, 2007; Shanghai constructs light transit system which covers most of the city area; Beijing builds more subways; and many cities increase the number of buses in services. Unfortunately, supply-side policies are not effective to reduce urban traffic congestion because urban commuting is subject to the theory of “triple convergences.” As Downs (2004) observed, in response to a capacity addition, three immediate effects occur. First, drivers using alternative routes begin to use the expanded roads. Second, those previously traveling during off-peak times (either immediately before or after the peak) shift to the peak (rescheduling behavior as defined previously). Third, public transport users shift to driving their vehicles. Because of the triple convergences and a potential huge induced demand, it is impossible to remove peak-hour

congestion from highways and roads by creating more road capacity.

In fact, transportation researchers have identified three traffic paradoxes showing that expanding a road system as a remedy to congestion is not only ineffective but also counterproductive under some conditions (Murchland, 1970; Arnott and Small, 1994; Braess et al., 2005). Specifically, the Pigou-Knight-Downs paradox states that adding extra road capacity to a road does not reduce travel time. The Downs-Thomson paradox states that the equilibrium speed of car traffic on the road network is determined by the average door-to-door speed of equivalent journeys by public transport. It follows that increasing road capacity can actually make overall congestion on the road worse. The Braess paradox states that adding extra capacity to a network, when the moving entities selfishly choose their route, can in some cases reduce overall performance and increase the total commuting time.

Traffic paradoxes exist because commuting generates negative externalities as a vehicle slows down all cars behind and adds to air pollution. Because of these negative externalities, in market equilibrium, drivers tend to drive more and thus there are more vehicles on roads, causing more congestion. In order to correct this market failure, a toll or road price needs to be levied on vehicles, so that traffic externality can be internalized and a social optimization will be reached. Therefore, it could be more effective to implement demand-side remedies to mitigate traffic congestion, because price mechanism not only affects driver's commuting behavior but also generates toll revenue for governments to provide better transportation network.

In this paper, by examining three traffic paradoxes, we will first prove that expanding a road system as a remedy to congestion is not only ineffective but often counterproductive. We then show that the paradoxes can be solved if congestion pricing is implemented. Our main contribution is to prove the three paradoxes and give social optimal solutions in their general

forms. To our knowledge, no previous study has attempted to do so, except Hartman (2007) which is for the Pigou-Knight-Downs paradox only. We hope that our theoretical analysis will shed important lights on transportation policy-making in China and other countries.

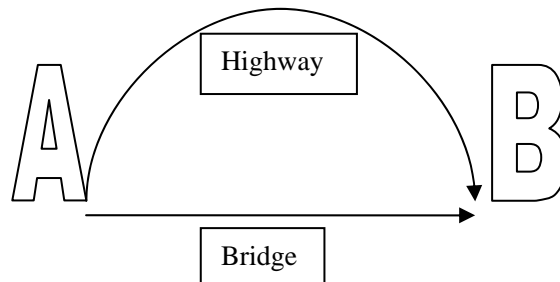
## 2. Traffic Paradoxes

This section discusses three traffic paradoxes, namely the Pigou-Knight-Downs paradox, the Downs-Thomson paradox, and the Braess paradox. They are paradoxes because expanding a road system as a remedy to congestion is not only ineffective but also counterproductive under some conditions. We will attempt to show these paradoxes in general cases in terms of their parameters.

### A. The Pigou-Knight-Downs paradox

The Pigou-Knight-Downs paradox states that adding extra road capacity to a road does not reduce travel time. This occurs because traffic may simply shift to the upgraded road from the other, making the upgraded road more congested.

**Figure 1: The Pigou-Knight-Downs Paradox**



In Figure 1, we assume that a bridge is added to the road system from A to B and the highway is always uncongested. The total travel flow is  $F$ , which is distributed between the

bridge ( $F_1$ ) and the highway ( $F_2$ ). The average travel time on the bridge ( $T_1$ ) is a linear function of the flow-to-capacity ratio and the average travel time on the uncongested highway ( $T_2$ ) is a constant. Hence, we have

$$T_1 = a + b \left( \frac{F_1}{C_1} \right); \quad T_2 = d; \quad F_1 + F_2 = F;$$

where  $a$ ,  $b$ , and  $d$  are positive parameters with  $d > a$ ;  $C_1$  is the traffic capacity of the bridge. In equilibrium,

$$T_1 = T_2.$$

We get

$$C_1 = \frac{bF_1}{d-a}.$$

At the boundary of  $F_1 = F$ , we obtain the boundary condition for  $C_1$

$$C_1(\text{boundary}) = \frac{bF}{d-a}$$

When  $C_1 < \frac{bF}{d-a}$ ,

$$\frac{dF_1}{dC_1} = \frac{d-a}{b} > 0,$$

meaning that increasing the bridge capacity will attract more drivers to use the bridge. However,

$$T_1 = T_2 = d,$$

indicating that increasing the bridge capacity will not reduce travel time. Therefore, the Pigou-

Knight-Downs paradox exists for any bridge capacity less than  $\frac{bF}{d-a}$ , because expanding the

bridge will only shift travelers to the bridge but not reduce travel time. The Pigou-Knight-Downs

paradox reveals that bridge expansion will not reduce travel time as long as there is at least one

person or car using the uncongested highway. Therefore, under the condition, bridge expansion

can not be justified for time saving.

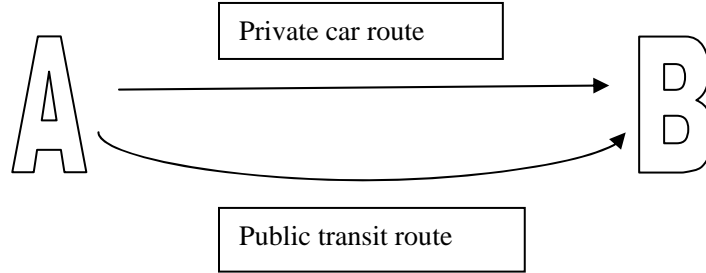
Using  $a=10$ ,  $b=10$ ,  $d=15$ , and  $F=1,000$ , Arnott and Small (1994) showed that the average travel time remains at 15 even if the bridge capacity continues to increase until it reaches 2,000,

which is twice as big as the total traffic volume, numerically proving the above Pigou-Knight-Downs paradox.

### **B. The Downs-Thomson Paradox**

The Downs-Thomson paradox states that the equilibrium speed of car traffic on the road network is determined by the average door-to-door speed of equivalent journeys by public transport. It follows that increasing road capacity can actually make overall congestion on the road worse. This occurs when the shift from public transport causes a disinvestment in the mode such that the operator either reduces frequency of service or raises fares to cover costs. This shifts additional passengers into cars. Ultimately, congestion on the road gets worse and the total commuting time increases.

**Figure 2: The Downs-Thomson Paradox**



In Figure 2, two routes connect A and B. One is private car route, with a traffic flow of  $F_1$ . The other is a public transit route, with a number of passengers of  $F_2$ . We assume that the average travel time on the private car route ( $T_1$ ) is a linear function of the flow-to-capacity ratio and the average travel time on the public transit route ( $T_2$ ) has a scale effect. Hence, we have

$$T_1 = a + b \left( \frac{F_1}{C_1} \right); \quad T_2 = d - F_2/e; \quad F_1 + F_2 = F;$$

where  $a$ ,  $b$ ,  $d$ , and  $e$  are positive parameters with  $d > a$ ;  $C_1$  is the traffic capacity of the private car route; and  $e$  tells the scale effect of the public transit. In equilibrium,

$$T_1 = T_2.$$

We get

$$F_1 = \frac{(de - ae - F)}{(be - C_1)} C_1, \quad \text{and} \quad T_1 = a + b \frac{(de - ae - F)}{(be - C_1)}.$$

At the boundary of  $F_1 = F$ , we obtain the boundary condition for  $C_1$

$$C_1(\text{boundary}) = bF/(d - a)$$

When  $C_1 \leq bF/(d - a)$ ,

$$\frac{dT_1}{dC_1} = b(de - ae - F)(be - C_1)^{-2} > 0, \quad \text{because } F \leq (d - a)e.$$

Therefore, increasing the traffic capacity of the private car route within the range of  $bF/(d - a)$  will increase traveling time on both routes ( $T_1 = T_2$ ), proving the Downs-Thomson paradox. The Downs-Thomson paradox suggests that highway expansion will be counterproductive under the

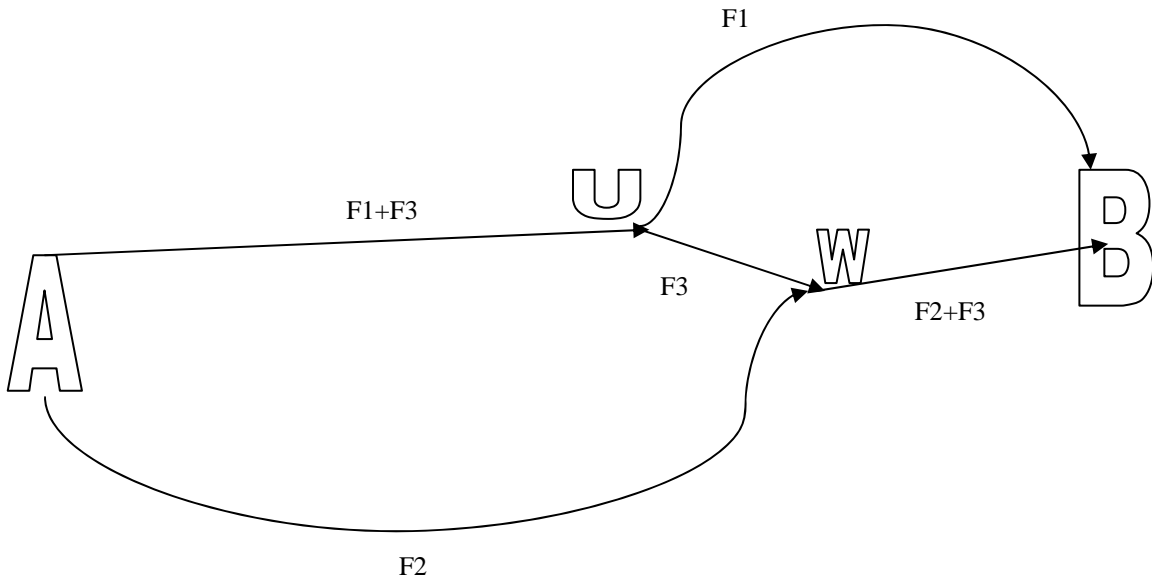
condition that there is at least one person using public transit that exhibits economic scale. The underlined logic behind this is the highway expansion will improve capacity, which in turn attracts more users from transit riders. A reduction in transit ridership will cause less frequency in transit operation and then longer commuting times for its users.

Using  $a=10$ ,  $b=10$ ,  $e=300$ ,  $d=20$ , and  $F=1,000$ , Arnott and Small (1994) found that the average travel time increases from 17.27 to 18.89 when the capacity of highway rises from 250 to 750, numerically proving the above Downs-Thomson paradox.

### **C. The Braess Paradox**

The Braess' paradox states that adding extra capacity to a network, when the moving entities selfishly choose their route, can in some cases reduce overall performance and increase the total commuting time.

**Figure 3: The Braess Paradox**



In Figure 3, two routes connect A and B before an uncongested causeway is added between U and W (hence  $F_3=0$ ), namely AUB and AWB and with traffic flow of  $F_1$  and  $F_2$ , respectively. The segments of AU and WB are congested and travel time on them increases proportionally with traffic flow. The segments of UB and AW are uncongested and travel time on them is assumed to be a constant ( $a$ ). Hence, we have

$$T_1 = a + \left(\frac{F_1}{e}\right); \quad T_2 = a + \left(\frac{F_2}{e}\right); \quad F_1 + F_2 = F;$$

where  $a$  and  $e$  are positive parameters. In equilibrium,

$$T_1 = T_2.$$

which gives traffic flow on each route (AUB or AWB) half of the total traffic flow and the average travel time without the causeway

$$T_{\text{without causeway}} = a + \frac{F}{2e}.$$

With the causeway added between U and W, three routes connect A and B, namely AUB, AUWB, and AWB (a fourth route of AWUB would be impractical). In this case, the traffic flow on AU is  $F_1 + F_3$ , the traffic flow on WB is  $F_2 + F_3$ , and the traffic flow on the causeway is  $F_3$ . Travel time on the uncongested causeway is assumed to be a constant ( $k$ , with  $k < a$ ). In equilibrium,

$$T_1 = T_2 = T_3,$$

where

$$T_1 = a + \frac{F_1 + F_3}{e}; \quad T_2 = a + \frac{F_2 + F_3}{e}; \quad T_3 = k + \frac{F_1 + F_3}{e} + \frac{F_2 + F_3}{e},$$

$$F_1 + F_2 + F_3 = F.$$

We get the average travel time with the causeway and traffic flow distribution,

$$T_{\text{with causeway}} = 2a - k.$$

$$F_1 = F_2 = ke + F - ae; \quad F_3 = 2ae - 2ke - F$$

Therefore,

$$T_{\text{without causeway}} - T_{\text{with causeway}} = \frac{F}{2e} - a + k.$$

A traffic paradox exists if  $F < 2(a - k)e$ . In this case,  $\frac{F}{2e} - a + k < 0$ , meaning that adding the causeway between U and W will increase the average commute time. In other words, expanding capacity to a network can in some cases will reduce overall performance and increase the total commuting time. The Braess paradox implies that construction of new uncongested highway segment(s) connecting congested highways will not alienate overall traffic times this newly constructed highway attracts users from uncongested highways.

Using  $a=20$ ,  $k=10$ ,  $e=100$ , and  $F=1,500$ , Arnott and Small (1994) found that the average

travel time increases from 27.5 without a causeway to 30 with a causeway, proving the Braess traffic paradox.

### 3. Congestion Pricing as a Resolution to Traffic Paradoxes

In the above analysis, commuters make route and modal choices based on their commuting time, i.e, in equilibrium, the average commuting time is the same for different routes or modals. Put it differently, commuters ignore how much delay they cause on other travelers but only pay attention to how long it takes them to commute. Therefore, the equilibrium numbers of commuters for routes and modals are not social optimal. This can be shown theoretically below.

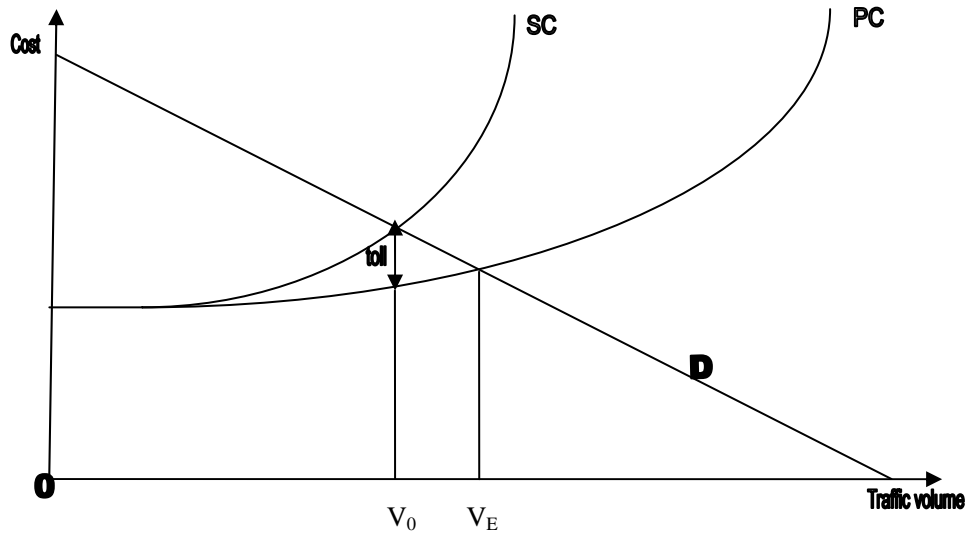
For each route or modal, let  $V$  be the traffic volume and  $t$  be the average commuting time. This gives the total commuting time  $tV$  and the marginal social cost,

$$SC = \frac{d(tV)}{dV} = t + V \frac{dt}{dV} = PC + EC$$

where  $PC$  (private average cost) is the average commuting time ( $t$ ) and  $EC$  is the externality cost ( $V \frac{dt}{dV}$ ). If the average commuting time increases with the number of commuters, like the case on congested urban roads,  $EC$  is positive and social marginal cost ( $SC$ ) will be higher than the private average cost ( $t$ ). Consequently, the equilibrium travel volume ( $V_E$ ) will be larger than the social optimal traffic volume ( $V_O$ ), i.e., too many commuters are on the roads. The former is determined based on the private average cost while the latter is calculated based on the social marginal cost, as shown in Figure 4. If the average commuting time decreases with the number of commuters, like the case of public transit,  $EC$  is negative and social marginal cost ( $SC$ ) will be lower than the private average cost ( $t$ ). Consequently, the equilibrium travel volume ( $V_E$ ) will be less than the social optimal traffic volume ( $V_O$ ), i.e., too few passengers are using public transit.

If the number of commuters does not affect the average commuting time, no congestion exists and externality disappears.

**Figure 4: Economics of Congestion Pricing**



To reach social optimization, externality should be internalized. In the case of congested urban roads, this suggests a toll of  $V \frac{dt}{dV}$  be charged on commuters. Because  $V \frac{dt}{dV}$  depends on traffic volume, the toll should be higher for more congested roads or periods than the one for less congested roads or periods. The optimal toll revenue equals to  $V^2 \frac{dt}{dV}$  and it is determined at  $V_0$ .

Many studies have discussed the theory and practice of congestion pricing. Evans (1992) examined when congestion pricing is a good policy. Giuliano (1992) assessed the political acceptability of congestion pricing. Small (1992, 1993) investigated toll revenues and spendings. Often, the public perceives toll simply as tax and commuters disliked the toll because

they find it coercive, in that they have few if any practical alternatives to paying the toll. Congestion pricing thus is considered as an economists' dream but politicians' nightmare.

However, in recent years, congestion pricing is becoming more popular in practice and receiving more public support. It also has been implemented in many cities in different countries. The best-known example of a successful congestion pricing program is the Area Licensing Scheme in Singapore where vehicles that wish to enter the central business district during peak hours must purchase a license (Watson and Holland, 1978; Phang and Toh, 2004; Decorla-Souza, 2006). In Spring 1998, the city shifted to a fully automated electronic charging system, with in-vehicle devices allowing payment by smart card, and enforcement using cameras and license plate reading equipment. The system has reduced traffic by 13 percent and increased vehicle speed by 22 percent. On February 17, 2003, London implemented a plan for using road pricing to combat congestion in central London. The scheme involves a standard per-day charge for vehicles traveling within a zone bounded by an inner ring road. The congestion charge, together with improvements in public transit financed with revenues from the charging system, led to a 15 percent reduction in traffic in central London. Travel delays have been reduced by 30 percent. Average traffic speed increased 37 percent. Excess waiting time on buses has fallen by around one-third (Decorla-Souza, 2006; Litman, 2006). In the first half year of 2006, Stockholm took a trial on congestion pricing, which resulted in 22 percent drop in vehicle trips and 9 percent increase in ridership on inner-city bus routes. Traffic accidents involving injuries fell by 5 to 10 percent. Exhaust emissions decreased by 14 percent in the inner-city. Residents of the City of Stockholm voted for continuation of the system in a referendum on September 17, 2006. The system was reinstated in 2007 (Decorla-Souza, 2006). Congestion pricing also has been implemented in the USA. Examples include the HOT lanes on I-15 in San Diego, California and

the bridge pricing in Lee County, Florida, both started in 1998. A better known example is the four variably-priced express lanes in the median of the State Route 91 Freeway in Southern California. Opened in December 1995, priced express lane each carry almost twice as many vehicles per lane than the free lanes during the peak hours, because of the effect of severe congestion on vehicle throughout in the free lanes (Harrington et al., 1998; Decorla-Souza, 2006).

In the following discussion, we will determine traffic volumes for routes and modals by minimizing the social total cost. If we let the social marginal cost be the same for different routes, the same solutions will be obtained. For all three traffic paradoxes discussed in section 2, our solutions show that increasing traffic capacity will always decrease the social total cost. Therefore, these paradoxes disappear if commuting externality is internalized and an optimal congestion pricing is implemented.

### A. The Pigou-Knight-Downs paradox

As specified earlier for the Pigou-Knight-Downs paradox, we have

$$T_1 = a + b \left( \frac{F_1}{C_1} \right); \quad T_2 = d; \quad F_1 + F_2 = F;$$

where  $a$ ,  $b$ , and  $d$  are positive parameters with  $d > a$ ;  $C_1$  is the traffic capacity of the bridge. Accordingly, the total cost can be written as a function of  $F_1$  after considering the flow constraint,

$$TC = T_1 F_1 + T_2 F_2 = dF + (a - d)F_1 + \frac{b}{C_1} F_1^2$$

Minimizing the total cost with respect to  $F_1$ , we get the following social optimal traffic flow on the bridge, the social total cost, and the relationship between the social total cost and the

bridge capacity.

$$F_1 = \frac{d-a}{2b} C_1$$

$$TC = Fd - \frac{(d-a)^2}{4b} C_1$$

$$\frac{dTC}{dC_1} = -\frac{(d-a)^2}{4b} < 0$$

The above solutions give two conclusions. First, the social optimal travel flow is the half of the equilibrium flow derived in section 2, showing that the bridge will be over-used in equilibrium when travelers choose routes based their own private cost. Second, the total travel cost decreases with the bridge capacity. Hence, the Pigou-Knight-Downs paradox disappears when traffic flows are distributed by minimizing the social total cost.

At the social optimum, the difference between the social marginal cost and the average private cost, i.e., the traffic externality, will be  $bF_1/C_1$  evaluated at the social optimal traffic flow. This determines the optimal toll and toll revenue on the bridge, which equal to

$$Toll = \frac{d-a}{2}$$

$$Revenue = \frac{(d-a)^2 C_1}{2b}$$

Therefore, to reach a social optimal solution and solve the Pigou-Knight-Downs traffic paradox, a toll of  $(d-a)/2$  should be charged on every commuter who is using the bridge. By charging such a toll, travel externality will be internalized and some of the commuters will be discouraged to use the bridge, making the road network efficient.

## **B. The Downs-Thomson paradox**

As specified earlier for the Downs-Thomson paradox, we have

$$T_1 = a + b \left( \frac{F_1}{C_1} \right); \quad T_2 = d - F_2/e; \quad F_1 + F_2 = F;$$

where  $a$ ,  $b$ ,  $d$ , and  $e$  are positive parameters with  $d > a$ ;  $C_1$  is the traffic capacity of the private car route; and  $e$  tells the scale effect of the public transit. Accordingly, the total cost can be written as a function of  $F_1$  after considering the flow constraint,

$$TC = T_1 F_1 + T_2 F_2 = dF - \frac{F^2}{e} + \left( a + \frac{2F}{e} - d \right) F_1 + \left( \frac{b}{C_1} - \frac{1}{e} \right) F_1^2$$

Minimizing the total cost with respect to  $F_1$ , we get the following social optimal travel flow on the bridge, the social total cost, and the relationship between the social total cost and the capacity of the private car route.

$$F_1 = \frac{de - ae - 2F}{2(be - C_1)} C_1$$

$$TC = Fd - \frac{F^2}{e} - \frac{(de - ae - 2F)^2}{4e(be - C_1)} C_1$$

$$\frac{dTC}{dC_1} = - \frac{b(de - ae - 2F)^2}{4(be - C_1)^2} < 0$$

The above solutions give two conclusions. First, because  $d > a$ , we can easily prove that the social optimal travel flow on the private car route is smaller than the equilibrium flow derived in section 2, showing that the private car route will be over-used if route choices are made based on private cost. Second, the total travel cost decreases with the capacity of the private car route. Hence, the Downs-Thomson paradox disappears when traffic flows are distributed by minimizing the social total cost.

At the social optimum, the difference between the social marginal cost and the average private cost, i.e., the traffic externality, will be  $bF_1/C_1$  evaluated at the social optimal traffic flow. This determines the optimal toll and toll revenue on the private car route, which equal to

$$Toll = \frac{bF_1}{C_1} = \frac{b(de - ae - 2F)}{2(be - C_1)}$$

$$Revenue = \frac{bF_1^2}{C_1} = \frac{bC_1(de - ae - 2F)^2}{4(be - C_1)^2}$$

Therefore, to reach a social optimal solution and solve the Downs-Thomson traffic paradox, a toll of the above amount should be charged on every commuter who is using the private car route. By charging such a toll, travel externality will be internalized and some of the commuters will be discouraged to use the private car route, making the transportation system efficient.

### C. Baress paradox

Before the causeway is added, each route (AUB or AWB) sees half of the total traffic flow, and the total social cost is

$$TC_{withoutcauseway} = aF + \frac{F^2}{2e}$$

With the causeway added, as specified earlier, we have

$$T_1 = a + \frac{F_1 + F_2}{e}; \quad T_2 = a + \frac{F_1 + F_2}{e}; \quad T_3 = k + \frac{F_1 + F_2}{e} + \frac{F_3 + F_3}{e};$$

$$F_1 + F_2 + F_3 = F.$$

Accordingly, the total cost can be written as a function of  $F_1$  and  $F_2$  after considering the flow constraint,

$$TC = T_1F_1 + T_2F_2 + T_3F_3 = (k + \frac{2F}{e})F - (k + \frac{2F}{e} - a)F_1 - (k + \frac{2F}{e} - a)F_2 + \frac{F_1^2}{e} + \frac{F_2^2}{e}$$

Minimizing the total cost with respect to  $F_1$  and  $F_2$ , we get the following social optimal travel flows and the social total cost.

$$F_1 = F_2 = \frac{ke + 2F - ae}{2}; \quad F_3 = ae - ke - F$$

$$TC_{withcauseway} = \frac{2keF - (a - k)^2 e^2}{2e} = 4(a - k)eF$$

Comparing with the equilibrium traffic flows determined by using the average private cost,  $F_3$  is smaller under the social optimization, suggesting that fewer travelers are using the causeway and the congested segments at the both ends. Also, we can prove that the total social cost will be lower after the causeway is added, because

$$TC_{withoutcauseway} - TC_{withcauseway} = \frac{(ae - ke - F)^2}{2e} > 0.$$

Therefore, when in social optimal solutions, adding causeway will reduce the total travel cost, indicating that the Braess paradox is solved.

At the social optimum, the difference between the social marginal cost and the average private cost, i.e., the traffic externality, will be  $(F_1 + F_3)/e$  on AU and  $(F_2 + F_3)/e$  on WB, both valuated at the social optimal traffic flow. This determines the optimal toll and toll revenue on the congested roads (AU or WB), which equal to

$$Toll = \frac{a - k}{2}$$

$$Revenue = \frac{(a - k)^2 e}{4}$$

Therefore, to reach a social optimal solution and solve the Braess traffic paradox, a toll of the above amount should be charged on every commuter who is using the causeway and thus making both segments of AU and WB more congested. By charging such a toll, travel externality will be internalized and some of the commuters will be discouraged to use the causeway, making the transportation system efficient.

#### 4. Discussions and Conclusions

This paper has illustrated that highway investment and expansion may not always help to mitigate traffic congestions and reduce travel time as intuitively suggested. Specifically, we presented three cases under which highway expansion or construction cannot reduce or even increase travel time by attracting more users from uncongested route or public transit. First, as the Pigou-Knight-Downs paradox states, adding a bridge to a road system does not reduce travel time. This occurs because traffic may simply shift to the bridge from the uncongested route, making the bridge more congested. Second, as the Downs-Thompson paradox states, increasing road capacity on a private car route can actually make overall congestion on the road worse. This occurs when commuters shift from public transport that exhibits economies of scale to private car that causes negative traffic externality. Third, as the Braess paradox states, adding extra capacity to a network, when the moving entities selfishly choose their route, can in some cases reduce overall performance and increase the total commuting time. In all three cases, commuters make decisions based on their private average cost and in equilibrium all routes or modes have the same average travel time.

The paper has shown that the above traffic paradoxes can be solved if traffic externality is internalized and commuters are distributed by minimizing the total social travel cost. Put it differently, adding road capacity will reduce commuting cost when travel decisions are made based on marginal social cost instead of private average cost. In order to reach the social optimal, a toll needs to be charged on commuters who are travel on congested routes. The amount of toll is determined by the difference between social marginal cost and private average cost at the social optimal traffic volume. For all three cases discussed above, this papers has derived the optimal tolls and toll revenues.

Strong policy implications can be drawn. First of all, what Pigou-Knight-Downs paradox

really means is that a construction of new highway segment under uncongested transportation network or an expansion of congested segments with uncongested alternative routes contributes little to traffic time reduction under the equilibrium condition. Therefore, expansion of uncongested highway network is economically unjustified. However, congestion pricing will produce different results. Unlike equilibrium choice, congestion pricing will justify an expansion of congested segments for traffic time reduction. This is because congestion pricing yields optimal allocation of traffics by internalizing congestion externality. Secondly, what Downs-Thomson paradox really means is that under equilibrium condition, highway construction will cause more traffic times by realigning traffic distribution between highway users and transit riders as long as there are at least one people uses transit. Therefore, highway expansion to promote private car uses is an inefficient strategy. Again, congestion pricing can justify highway expansion that will reduce traffic times under optimal allocation of traffic volume. Thus, highway expansion, transit development and pricing should all be integrated parts of transportation development strategy. Finally, what Braess paradox really suggests is that construction of highway segments should be carefully determined in terms of layouts and connectivity. Otherwise, highway construction may not mitigate traffic condition under the equilibrium condition. Congestion pricing, like the above two cases, will ensure additional road capacity to reduce travel time. It should be pointed that policy implications of our analyses should be cautiously interpreted, mainly because that traffic time, instead of costs, is used in our analysis. This is particularly important for traffic allocation between private car users and public transit riders since costs are primary consideration for transit users.

Finally, this paper has demonstrated the need for comprehensive transportation development and planning strategy that should include highway expansion, new construction,

public transit users and pricing mechanism to better address urban traffic. Solely relying on highway construction may be counter-productive in terms of traffic congestion. Often, it is more effective if policies are made by considering commuter economic behavior and using pricing mechanism. To mitigate urban traffic congestion, both supply-side and demand-side policies need to work together.

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