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**Studying Economic Growth:
An Avenue for Enhancing Student Empirical Skills**

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Abstract

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Studying Economic Growth: An Avenue for Enhancing Student Empirical Skills

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Abstract

The study of economic growth provides an opportunity for students to exercise their empirical skills, reinforcing the tool building that occurs in statistics and math courses. Descriptive analysis allows lower level students to develop their ability to work with data as they examine how fast the economy has grown, ascertain the regularity versus irregularity of this growth, test whether the U.S. economy is slowing down, and perform simple extrapolation forecasts. Explanatory analysis allows higher level students in macro and econometric courses to see how theory and empirics can complement each other, and see why econometric issues matter, as they seek estimates for parameters consistent with the theory. The activities presented here may be of interest to those seeking to enhance the teaching of analytical skills “across the curriculum.”

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1. Introduction

In the opening two paragraphs of his *Wealth of Nations*, Adam Smith (1776 [1993], p.8) emphasizes the importance of understanding economic growth, making the basic point that a nation's wealth comes from production. He identifies labor as the fundamental productive input, and later presents a whole "book" (Book II) emphasizing capital. The work of Solow (1956) and Swan (1956) led to the standard growth theory model, which captures the roles played by labor, capital, and technology. The resurgent research interest in growth theory (e.g., Romer (1986), Lucas (1988)), which has focused on explaining technical change, has been accompanied by an increased focus on economic growth in the typical macroeconomics course.¹ Studying economic growth presents opportunities for enhancing student empirical skills, reinforcing skills learned in tool building courses like statistics. This paper presents a series of activities found to be useful in macroeconomics courses ranging from the introductory level to first year graduate level, and useful in an introductory econometrics course.

Why should students in a macroeconomics course spend time working with data when there is more theory than can be covered? First, relating theory to data tends to make the student more interested in the theory. Learning may suffer when theory and empirical tools are presented independently because students may fail to see the usefulness of either. Theory and empirics complement each other in the pursuit of understanding economic growth, and experiencing this can enliven student interest. Second, including some work with data in a macroeconomics course reinforces skills that are particularly marketable. Instructors in departments interested in teaching empirical and analytical skills across the economics curriculum should find value in the activities presented here.

Simple descriptive empirical analysis can be presented in an introductory macroeconomics course. This promotes knowledge of economic facts, as it promotes technical skills with practical value. The facts tend to stick because students learn them by their own hand. As she learns that the U.S. economy grows at about 3.4 percent per year, a college freshman also learns a method she can also use to measure the rate of return of her own future stock portfolio. Without proceeding to regression analysis, students can gain useful experience

¹ See R.G.D. Allen (1969), Barro and Sala-I-Martin (1995), and Romer (2006) for excellent discussions of growth theory meant for beginning graduate students or well prepared undergraduate students.

using a spreadsheet by being asked to constructing a model that “fits” the data, and they are often excited to see that they can use a simple model to forecast the future. More advanced macro students, or students in econometrics classes, can apply regression analysis and more advanced descriptive modeling, including use of a dummy variable, as they seek to understand whether or not the U.S. economy is slowing down.

An explanatory analysis of economic growth allows more advanced macro students and econometric students to sharpen their empirical and theoretical skills in tandem. Estimating a production function, for example, naturally leads to a discussion of the microfoundation of macroeconomics, and a review of the theory of the firm. Students will find that a model with no technical change does not well explain the growth experience of the U.S.. The search for a model that is theoretically sensible leads to some interesting theoretical and empirical issues.

The activities presented here have been used at the University of Nevada, Reno, which is a mid-sized, state university. The most simple activities have been used in introductory and intermediate level macro-economics courses. The more advanced have been used in intermediate and master’s level macroeconomics courses, and in an introductory econometrics course. All of the statistical procedures presented here are simple enough that they can be performed using a spreadsheet program, which facilitates their integration into courses other than econometrics.

A subset of these activities can be combined to form a “project” that tells a story of the U.S. economy. When projects have been assigned, students have been given a suggested structure. However, students are also encouraged to be creative, earning additional credit for effectively pursuing questions that have not been assigned. The project is a product the student produces in the course and can take with them. More than one student has returned after graduation to say that they have obtained a job specifically because they were able to present their project to a prospective employer as an example of what they could do in terms of working with data and assembling a report.

This paper is organized as follows. Illustrations of descriptive analysis are presented in section 2, ranging from the most simple to more complex. In section 3, illustrations of explanatory analysis are presented, again ranging from the most simple to the more complex. The final section offers some concluding remarks.

2. Descriptive Analysis

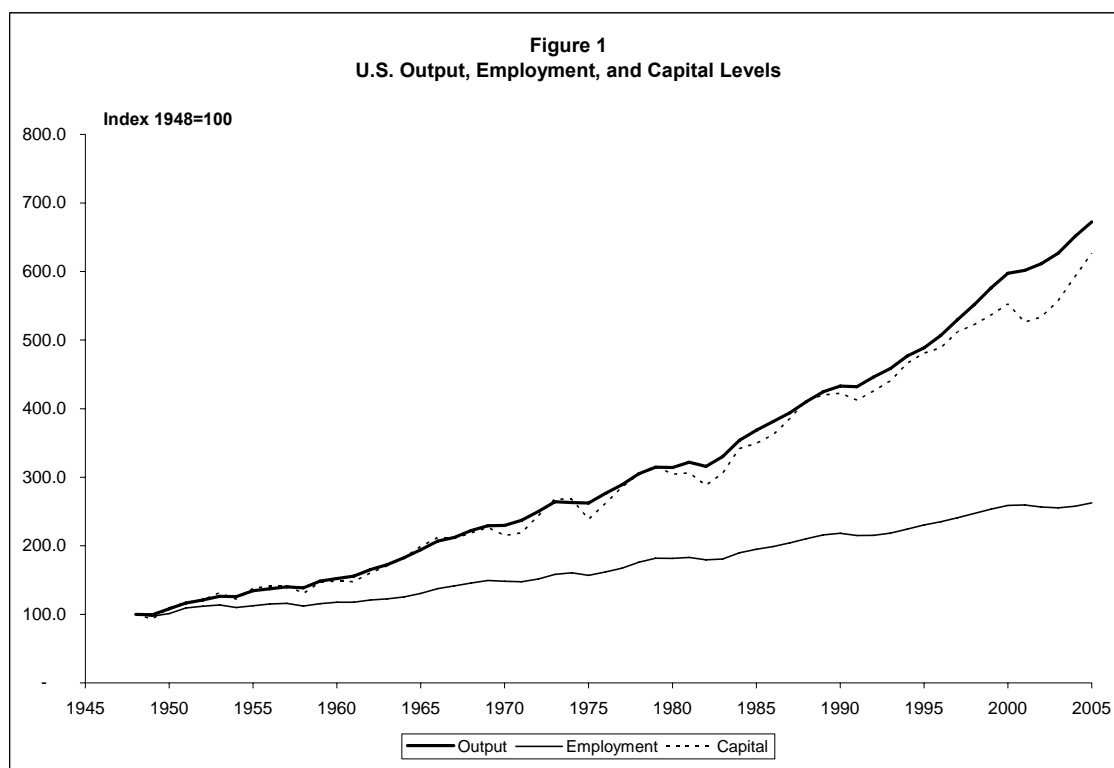
In this section, we present data on U.S. output, employment, and capital for the 1948-2005 period.² Using the output measure, we illustrate a variety of descriptive analyses that can be used to characterize what has happened historically.

The Bureau of Economic Analysis within the U.S. Department of Commerce is responsible for measuring the U.S. economy's output of goods and services. Their real gross domestic product statistic (GDP) is the most commonly used output measure. This can be downloaded from their National Income and Product Accounts website (U.S. Department of Commerce, 2005a). The most common measure of employment is the civilian employment measure provided by the U.S. Bureau of Labor Statistics. However, this employee count includes both part-time and full-time employees, meaning a shift toward part-time work over a given period of time could result in an increase in employment but a decrease in total labor hours. For this reason, we choose to use the full-time equivalent employees labor measure provided by the U.S. Department of Commerce (2005b) Bureau of Economic Analysis. The U.S. Department of Commerce (2005c) provides a series of capital stock measures, and we chose here to use a private sector measure that includes private fixed capital and durable consumption goods. At any given time, some portion of the stock of productive capital is idle. It is common (see Intriligator, 1978, p. 263) to multiply a capital measure by a utilization rate and use the resulting adjusted capital stock measure, so the capital variable more effectively represents the amount of capital employed. We adjust the capital measure presented here using the manufacturing capacity utilization rate provided in the Economic Report of the President (2007).

To visually compare output, employment, and capital levels over time, it is useful to plot each of these statistics in the same diagram. However, when the value of one statistic is much larger than others, the large statistic will tend to dominate the diagram making the plot rather

² An output measure is available for the years 1929-2006. However, the 1948-2005 period has been chosen because full time equivalent employee data is only available for the years 1929-2005, and capital data is available for the years 1948-2006.

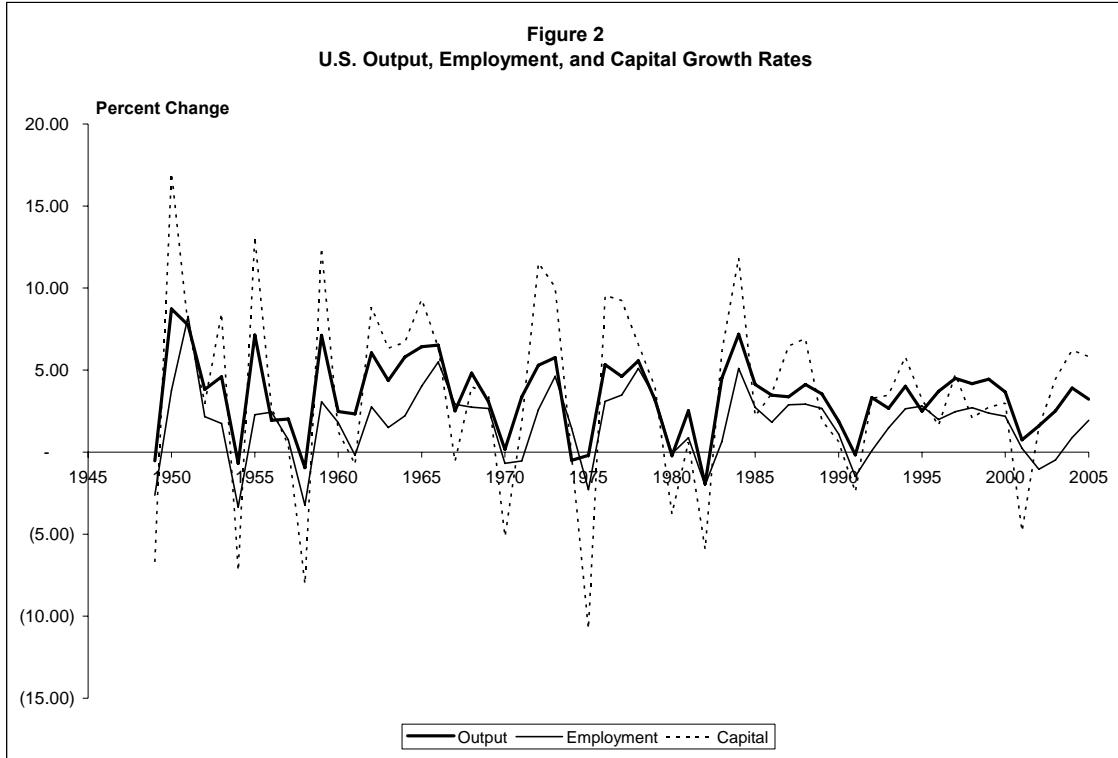
uninformative. It is good for student to experience this first hand, and then to learn this problem can be avoided if each statistic is converted to an index. Figure 1 presents the levels of output, employment, and capital for the U.S. using indexes. We choose to construct indexes so 1948 is the “base year.” Here, students can be taught that it is typical for an index to be set equal to 100 in the base year, and be taught that an index is constructed so the percentage change in the index from one year to the next is the same as the percentage change in the raw data series. By plotting these indexes in the same diagram, we obtain a snapshot of growth of each data series in an absolute and relative sense. In Figure 1, we see output and capital grow at about the same rate, each growing about six-fold over the period, while employment grows more slowly, not quite a three-fold increase. We also can see that the atypical periods when output decreases are positively correlated with decreases in capital and employment.



At what rates do output, employment, and capital grow? This is a convenient point to teach introductory level students how to calculate an average annual growth rate using the internal rate of return concept. Let r denote the average annual growth rate. Let PV denote the “present value” of the data series, and let FV denote the “future value,” where it is understood

that the present comes before the future. Let n denote the number of compounding periods, or in our case the number of years, that it takes for the present value to grow into the future value. These four variables are related by the formula $FV = PV[1 + r]^n$, arguably the most important formula used in finance.

Given numbers for any three of these variables, one can calculate the value of the unknown fourth variable using the formula. As an example, real GDP for the U.S. in 1948, given in year 2000 dollars, was \$1,643.2 billion. In 2005, it was \$11,048.6 billion. In the formula, we can thus set $PV = 1643.2$, $FV = 11,048.6$, $n = 57$. Using some algebra, we can solve for the average growth rate $r = [FV / PV]^{1/n} - 1$ and calculate $r = 0.0340$. Thus, we find that U.S. output grew at an average annual rate of 3.40 percent over the 1948-2005 period. Using the same method, we find that capital grew at an average annual rate of 3.27 percent, and employment grew at 1.70 percent. This method of characterizing growth is convenient because it requires only two data points and a handheld calculator, a practical method students could one day use to keep track of the rate of growth of a stock portfolio or home price.



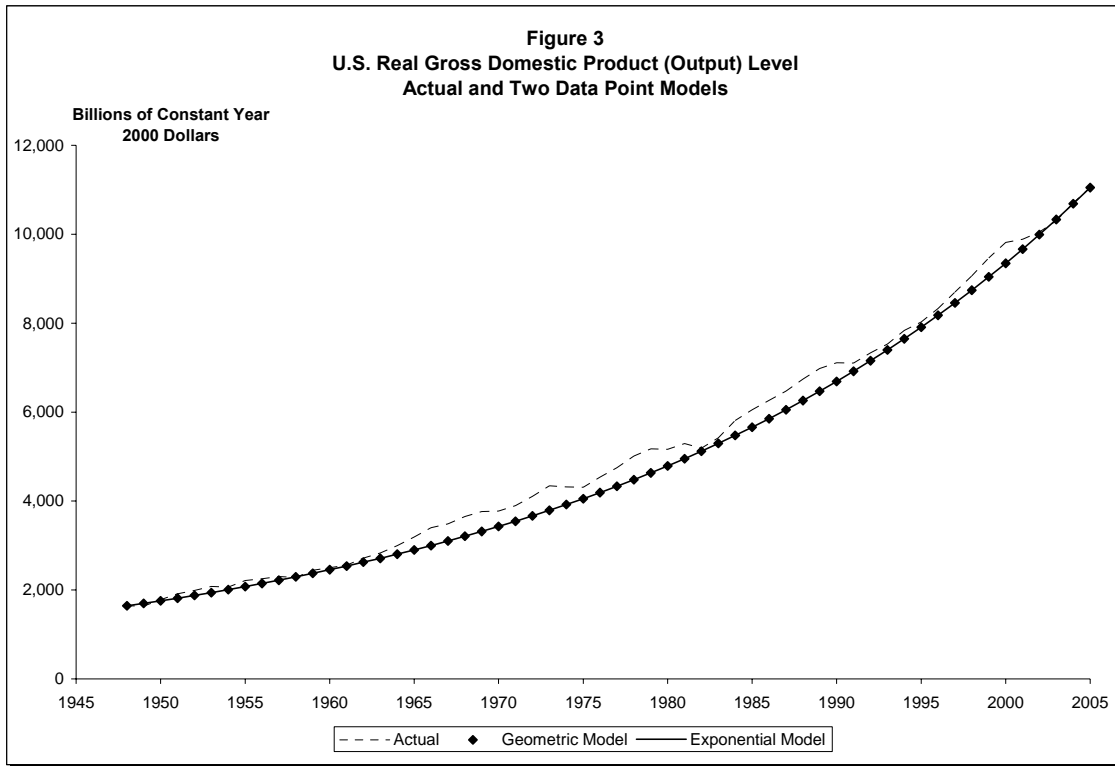
Examining Figure 1, one can see the growth rates of output, employment, and capital are not constant over time. Figure 2 presents the year over year growth rates for output, employment, and capital. This provides information not so evident in the plot of the levels. Even though output and capital grow at about the same rate, we see capital growth is much more volatile. It is also more apparent in Figure 2 that the growth rates of output, employment, and capital are positively correlated.

The path followed by a data series over time can be modeled. Let \hat{Y}_t denote the “predicted” value of a variable, either during period t or at point in time t . The descriptive model relates \hat{Y}_t to t . Visually, Figure 2 suggests that output grows at a rate that is roughly constant over longer periods of time, and the same for labor and capital. The geometric model, $\hat{Y}_t = \hat{Y}_0 [1 + r]^t$, and exponential model, $\hat{Y}_t = \hat{Y}_0 e^{rt}$, each capture growth at a constant rate. Introducing each of these models exposes students to the difference between discrete time, where time is broken into periods, and continuous time where it is not.

Introductory macro students can construct a constant growth model using Excel and some algebra. Having obtained an average annual real GDP growth rate of $r = 0.0340$ using the $FV = PV[1 + r]^n$ formula, as shown above, and recognizing the initial 1948 value of output as 1,643.2, the introductory student can readily be taught to construct the geometric model $\hat{Y}_t = 1,648.2[1 + 0.0340]^t$ in an excel column, and plot it along with the actual real GDP level as demonstrated in Figure 3.

A constant growth rate can also be obtained with just two data points using the exponential model, and doing so gives students practice using the natural log. Taking the natural log of $\hat{Y}_t = \hat{Y}_0 e^{rt}$ yields $\ln(\hat{Y}_t) = \ln(\hat{Y}_0) + rt$, and solving for r we obtain $r = [\ln(\hat{Y}_t) - \ln(\hat{Y}_0)]/t$. Letting $t = 57$ in this last condition and entering the values $\hat{Y}_0 = 1,643.2$ and $\hat{Y}_{57} = 11,048.6$, we obtain a continuously compounded growth rate of $r = 0.0334$. Using excel to plot the model, a student can see that the exponential model $\hat{Y}_t = 1,634.2e^{0.0334t}$ with the lower growth rate produces the same path when plotted as the geometric model $\hat{Y}_t = 1,648.2[1 + 0.0340]^t$ with the higher growth rate, which naturally stimulates a discussion of the difference between annual and

continuous compounding. We distinguish the two models in Figure 3 by using markers for the plot of the geometric model and a continuous curve for the plot of the exponential model.³



The two data point geometric and exponential models are the most simple way to model a constant growth rate trend, but they are not best fit models. In an intermediate macroeconomics course, or econometrics course, ordinary least squares regression procedures can be reinforced by having students estimate a constant growth model that best fits the trend.

To apply linear regression, the parameters to be estimated must bear a linear relationship to the dependent variable. In the exponential model, the variable r is an exponent, so least squares cannot be directly applied. However, this gives opportunity to expose students to the “transformation of variables technique.” While there is a nonlinear relationship between \hat{Y}_t and

³ A challenge that can be given to students is to show the algebraic relationship between the constant geometric growth rate r_g along this model path and the constant exponential growth rate r_e . The models $\hat{Y}_t = \hat{Y}_0 [1 + r_g]^t$, and $\hat{Y}_t = \hat{Y}_0 e^{r_e t}$ imply $[1 + r_g]^t = e^{r_e t}$. Taking the natural log, we obtain, $\ln[1 + r_g] = r_e$, which gives r_e as it depends upon r_g . Taking the exponential of the last equation, and solving for r_g , we obtain $r_g = e^{r_e} - 1$, which gives r_g as it depends upon r_e .

r in equation $\hat{Y}_t = \hat{Y}_0 e^{rt}$, there is a linear relationship between $\ln \hat{Y}_t$ and r in the equation $\ln \hat{Y}_t = \ln \hat{Y}_0 + rt$, which is obtained after taking the natural log. In Excel, the variable $\ln Y_t$ can be created from the variable Y_t , which is the column of real GDP data. Excel can be used to regress $\ln Y_t$ on t , where the variable t is a column of data beginning with 0 in 1948 and ending at 57 in 2005. The results of the regression can be presented as

$$(1) \quad \ln \hat{Y}_t = 7.47 + 0.0331t, \quad R^2 = .9949.$$

(0.011)*** (0.0003)***

The path for \hat{Y}_t , for times $t = 0, 1, \dots, 57$, is given by $\hat{Y}_t = 1749.4e^{0.0331t}$, obtained by taking the exponential of equation (1). This path is plotted in Figure 4 below.

Having intermediate macro students work with data in this way provides them with the opportunity to practice presenting regression results in an informative and professional manner, reinforcing what they learned in statistics or econometrics. Here, the standard error of each estimate is shown in parenthesis underneath the estimate. As is also a common practice, the significance of each estimate is indicated using asterisks. When a result like this is first presented, it is a good opportunity to remind students that a t-statistic is a test statistic (equal to the coefficient estimate divided by the coefficient estimate standard error) that can be used to evaluate the “null hypothesis” that the coefficient’s true value is equal to zero. In this case, the interesting null hypothesis is that the U.S. economy is not growing (i.e., $r = 0$). The three asterisks on the associated standard error indicate that we can reject this null hypothesis at the 1 percent “level of significance.” Students can be reminded that this implies the “confidence level” we have in rejecting this null hypothesis is 99 percent, or more. Two asterisks would indicate significance with a 5 percent level of significance, or 95 percent confidence, but not with a 1 percent level. One asterisk would indicate significance with a 10 percent level of significance, or 90 percent confidence, but not with a 5 percent level. The absence of asterisks would indicate that we cannot reject the null hypothesis with 90 percent confidence. Students can be reminded

to look at the “p-value” to get an indication of how far the coefficient is from meeting the significance threshold. The R^2 and $adjR^2$ as measures of fit can also be reviewed.⁴

The polynomial model is another standard model that can be used to characterize a trend. In this model, the predicted value \hat{Y}_t is directly related to the time variable t . An example is the third degree polynomial model $\hat{Y}_t = a_0 + a_1t + a_2t^2 + a_3t^3$. The parameters a_0, a_1, a_2 , and a_3 can be estimated by regressing the real GDP variable Y_t on t, t^2 , and t^3 . Examining Figure 3, it is evident that real GDP increases over time. If only the variable t is included in the regression, then it is implicitly being assumed that there is a constant increment added to output each period. Including t^2 in the regression allows one to test this assumption, which is equivalent to seeing whether allowing for a nonlinear trend can improve the model’s fit. Adding t^3 to the regression is also of interest because this allows for an inflection point in the trend. For example, the trend can at first increase at an increasing rate but then increase at a decreasing rate. For the real GDP data we have, the estimated coefficient on t^3 is statistically significant, so all three time variables are left in the regression. The estimated model can be presented as

$$(2) \hat{Y}_t = 1,571.1 + 85.6t + 0.082t^2 + 0.023t^3, \quad R^2 = .9979 .$$

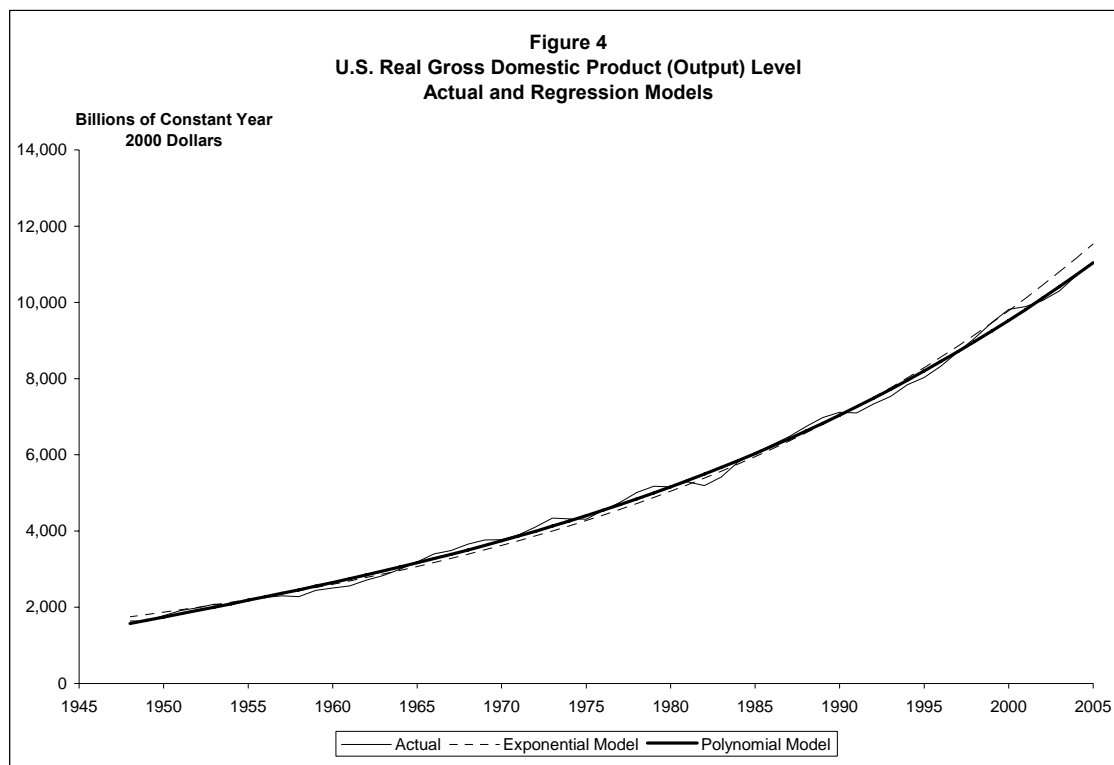
(62.9)*** (9.65)*** (0.40) (0.005)***

In Figure 4, the exponential and polynomial models obtained from regressions (1) and (2) are plotted along with actual real GDP. In contrast to the two data point constant growth rate exponential model, which in Figure 3 starts precisely at the beginning data point and ends precisely at the ending data point, the best fit constant growth rate model presented in Figure 4

⁴ An econometrics or intermediate macro student can also be reminded that the null hypothesis used in the statistical test need not be that the coefficient is zero. For example, we can test whether the estimated growth rate of $r = 0.0331$ obtained from the regression (1) is significantly different than the estimate $r = 0.0334$ obtained from two data point exponential model. In general, the test statistic $t(\text{degrees of freedom})$ is related to the least squares estimate $\hat{\beta}$, the null hypothesis β_{H_0} , and the coefficient estimated standard error $se(\hat{\beta})$ according to

$t(\text{degrees of freedom}) = [\hat{\beta} - \beta_{H_0}] / se(\hat{\beta})$. For our estimation (1), we find $t(56) = [.0334 - .0331] / .0003 = 1.059$. At a 10 percent level of significant, we would need a test statistic of 1.65 to reject the null hypothesis. Thus, we find we cannot reject the hypothesis that the growth rate obtained from the more complicated regression model is different from the growth rate obtained from the more simple two data point model.

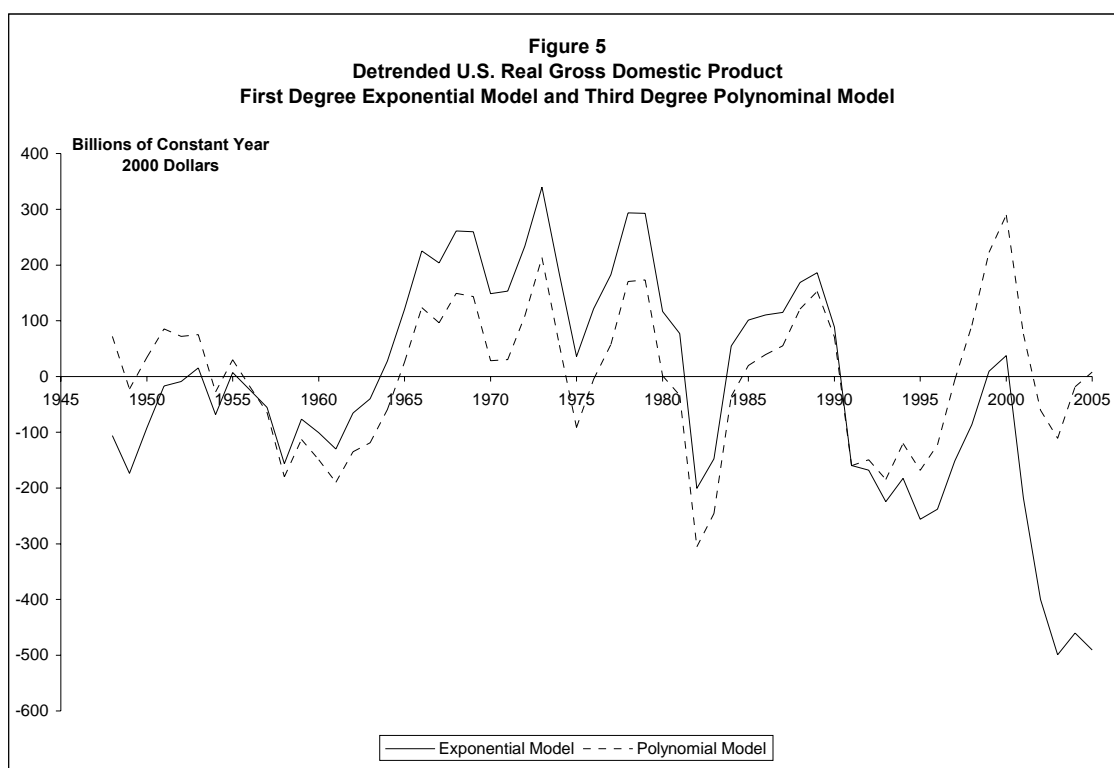
neither begins nor ends on the data points. One can also see in Figure 4 that the polynomial model better fits the data than the exponential model. This is because the inclusion of the higher order time variables allows the growth rate of real GDP to change in a flexible manner along the polynomial path, while it is restricted to a constant growth rate along the exponential path.



Traditional business cycle analysis involves decomposing real macroeconomic variables, like real GDP, into a deterministic trend component and a cyclical or non-trend component (See Enders, 1995, p. 181-183). A “detrended” data series is obtained by subtracting the estimated trend level from the actual data level. The equation $\hat{Y}_t = 1749.4e^{0.0331t}$ gives the trend value for real GDP for period t as predicted by the constant growth rate exponential model, while equation (2) gives it for the polynomial model. The detrended real GDP value for period t is given by $\varepsilon_t = Y_t - \hat{Y}_t$.

By plotting the detrended real GDP series, we get a representation the “business cycles” experienced by the U.S. economy. The detrended real GDP series obtained from the exponential and polynomial models are presented in Figure 5. A complete cycle involves going from the

depth of one “trough,” where the actual series is below trend, to a “peak,” where the actual series is above trend, and then back to another trough. Because the third degree polynomial model better fits the trend than the first degree exponential model, the polynomial model identifies more cycles. The polynomial model indicates there have been 7 business cycles since 1948, with successive trough low points (i.e., bad times) in 1949, 1954, 1961, 1975, 1982, 1993, and 2003. The successive peak high points (i.e., good times) are 1951, 1955, 1973, 1979, 1989, 2000, and 2005. The exponential model identifies 5 cycles, with trough lows in 1949, 1954, 1982, 1995, and 2005 and peak highs in 1953, 1955, 1973, 1989, and 2000.



At this point, in an intermediate macroeconomics or econometrics course, one can teach students the Nelson and Plosser (1982) critique of this traditional business cycle approach, exposing students to the idea that a trend can be “stochastic” rather than “deterministic.” When we find a trend by fitting the exponential and polynomial models to the data, we are implicitly assuming that the trend is deterministic. When a trend is assumed to be deterministic, any “shock” that causes a deviation from trend is “transitory,” meaning forces are automatically set in motion that will return the series to the trend so that the deviation dissipates. Examining Figure 5, this appears to be the case. However, Nelson and Plosser (1982) challenged this

traditional view by providing evidence that real GDP and other important macroeconomic time series more likely follow stochastic trends. When the trend is stochastic, the shocks are non-transitory, meaning they permanently change the trend. Because these random non-transitory shocks can be positive or negative, the observed stochastic trend can be visually indistinguishable from a deterministic trend that experience transitory shocks.

Below, we will show that the estimated exponential model (1) and the estimated polynomial model (2) can be used to forecast real GDP. As Enders (1995, p. 184) explains, the primary problem with using a deterministic model in this manner when the actual trend is stochastic is that the variance of the forecast error becomes infinitely large as time proceeds. Using an “augmented Dickey-Fuller test,” (which cannot be performed using Excel), we indeed find that we cannot reject the null hypothesis that real GDP series is “non-stationary,” an indication that trend followed by real GDP is stochastic rather than deterministic. Note that each detrended series in Figure 5 seems to exhibit an increasing variance as time unfolds. We expect to see this “heteroskedasticity” when a stochastic trend is modeled using a deterministic trend model. Knowing real GDP is a non-stationary series, the problem with forecasting real GDP using the exponential model (1) or polynomial model (2) is we expect the deviation of the model prediction from actual real GDP to increase as the future unfolds.⁵

Econometric problems like non-stationarity and serial correlation are often dealt with by “log differencing” the data. The natural log difference $\ln Y_t - \ln Y_{t-1}$ is one measure of the growth rate of Y_t , a measure that is between the standard percentage change, $[Y_t - Y_{t-1}]/Y_{t-1}$, and the non-standard percentage change $[Y_t - Y_{t-1}]/Y_t$. Using an augmented Dickey-Fuller test, we find that we can easily reject the null hypothesis that this real GDP growth rate series is “non-stationary.” This indicates we have more econometric confidence in a forecast of the real GDP growth rate that comes from a model that fits the real GDP growth rate than we would have in a forecast of the real GDP level coming from a model that fits the level.

Because a log difference is approximately equal to a growth rate, we can effectively model the path followed by a log difference data series using a growth rate model, and the

⁵ Perron (1989) provides an important critique of the Nelson and Plosser findings. In particular, he demonstrates that the non-stationarity finding can be attributed to just two important shocks, those that occurred in 1929 and 1973. Once these two shocks are accounted for using the assumption of a structural break, economic fluctuations appear to be stationary around a deterministic trend, as assumed in traditional business cycle analysis. This is an interesting and pedagogically useful issue for students in an econometrics class to explore.

exponential model is useful in this regard. Consider the second degree exponential model

$\hat{Y}_t = Y_0 e^{r_1 t + \frac{1}{2} r_2 t^2}$. This model can be used to test whether the rate of economic growth in the U.S.

is slowing over time. Taking the natural log we obtain $\ln \hat{Y}_t = \ln \hat{Y}_0 + r_1 t + \frac{1}{2} r_2 t^2$. Having

intermediate macro and econometrics students take the time derivative of this last equation provides the opportunity to teach the relationship between natural logarithms and growth rates.

Doing so, we obtain $\left[\frac{1}{\hat{Y}_t} \left[d\hat{Y}_t / dt \right] \right] = r_1 + r_2 t$. The left side of this equation represents the growth rate output. If r_2 is equal to zero, then this growth rate model indicates the growth rate is constant r_1 . However, if r_2 is significantly different than zero, then the growth rate is not constant.

Using the log difference $\ln Y_t - \ln Y_{t-1}$ to represent the growth rate $\left[\frac{1}{\hat{Y}_t} \left[d\hat{Y}_t / dt \right] \right]$, and regressing $\ln Y_t - \ln Y_{t-1}$ on t , we obtain

$$(3) \quad \ln Y_t - \ln Y_{t-1} = 0.039 - 0.0002t, \quad R^2 = .02106 .$$

(.006)*** (.0002)

Because the coefficient estimate $r_2 = -0.0002$ is not significantly different than zero, we do not find sufficient evidence here to reject the null hypothesis that the U.S. economy has grown at a constant rate. The best fit log difference model is a constant growth rate model, which is most simply found by taking the average of $\ln Y_t - \ln Y_{t-1}$ over the 1949-2005 period. Doing this, we find that the average rate of economic growth is $0.0334 = 3.34\%$.

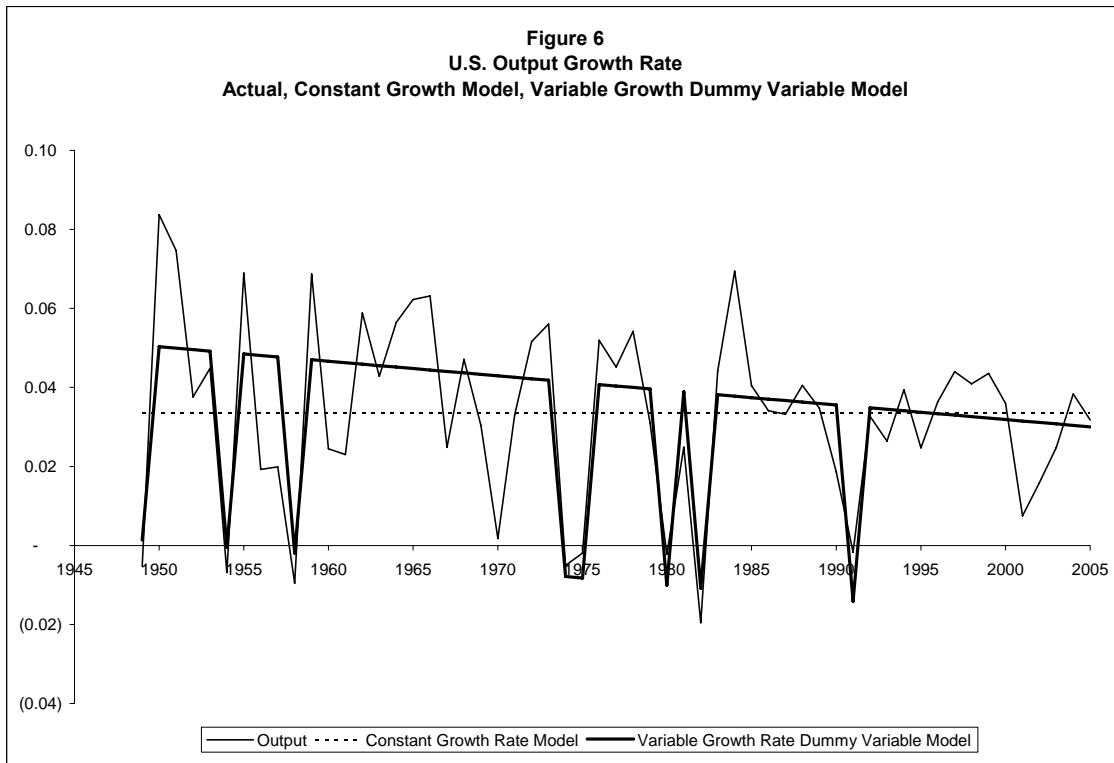
As can be seen in Figure 2, the recessionary years, where the real GDP growth rate is negative, particularly stand out. In an intermediate macroeconomic course or econometrics course, one can introduce the use of dummy variables by introducing a dummy variable for the recessionary years, demonstrating how the significance of a coefficient for a model can dramatically change when the overall error in the regression is reduced. Let D denote the value of the recession dummy variable, where $D = 0$ for all non-recessionary years and $D = 1$ for the recessionary years 1949, 1954, 1958, 1974, 1975, 1980, 1982, and 1991. Adding this dummy variable to the last regression and estimating the new model, we obtain

$$(4) \quad \ln Y_t - \ln Y_{t-1} = 0.051 - 0.0004t - 0.049D, \quad R^2 = .5669 .$$

(.004)*** (.0001)*** (0.006)***

Comparing equations (3) and (4), note that the addition of the dummy variable significantly increases the R^2 of the model. This significant reduction in the amount of unexplained error alters the significance of the coefficient on the time variable t . In contrast to regression (3), regression (4) indicates the rate of economic growth in the U.S. is decreasing over time in the amount of 0.04 percentage points per year.

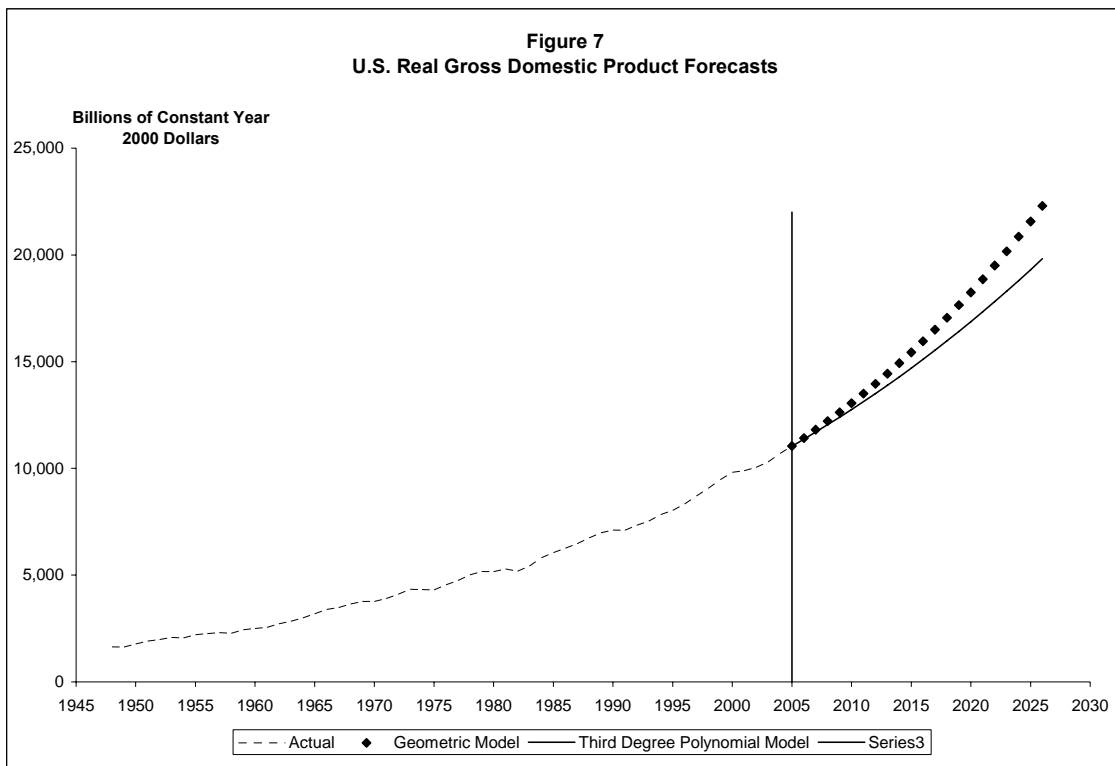
In Figure 6, the actual growth rate of real GDP is presented, measured as the log difference $\ln Y_t - \ln Y_{t-1}$, along with constant estimated constant growth rate of 3.34 percent and the decreasing growth rate path described by the dummy variable model (4). The dummy variable model indicates U.S. economy has been following a growth path such that the rate of economic growth has decreased from 5.1 percent in 1949 to 3.0 percent in 2005.

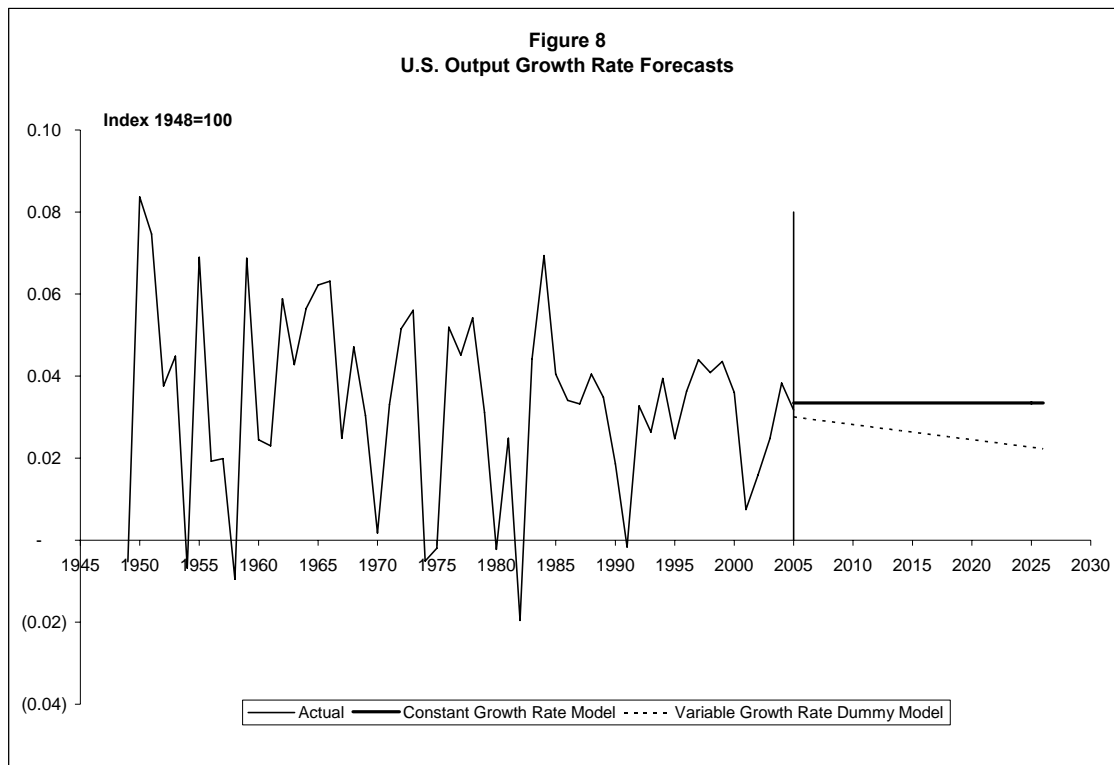


Models that characterize historical trends can be used to forecast the future. The forecast is generated by simply assuming the historical trend will continue. Figure 7 presents the 20 year 2006-2026 forecast of real GDP obtained from the two data point geometric model and the third degree polynomial model obtained from regression (2). An introductory level student can

generate the forecast from the two data point geometric model, while the polynomial model obtained from regression can be added to the task of a higher level student. The forecast provided by the geometric model is a path that maintains a constant growth rate of 3.40 percent compounded annually, or 3.31 percent compounded continually. The fit of the third degree polynomial model is associated with a declining growth rate, explaining why the forecast is not as optimistic.

Figure 8 presents two forecasted paths for the real GDP growth rate. The constant growth rate model assumes the historical average of 3.34 percent extends into the future. The dummy variable model assumes the growth rate decreases according to regression (4). The dummy variable model predicts the U.S. rate of economic growth will decrease from 3.31 percent in 2006 to 2.03 percent in 2026. At this point in the course, the implications of a constant versus declining growth rate for Social Security are interesting to discuss.





3. Explanatory Analysis

A growth theory is an explanation of why an economy's output level exhibits its observed pattern. Growth theories are typically presented without having students “get their hands dirty” with data. However, for many students, studying the theory becomes more meaningful when they can see how it relates to data. The process also helps students develop marketable econometric and modeling skills.

The neoclassical growth model, based on the seminal work of Solow (1956) and Swan (1956), postulates that output growth is the result of employment growth and capital accumulation in a constant returns to scale environment where labor and capital are imperfect substitutes. To effectively explain the path followed by the U.S. economy, technological improvement must also be added to the model.

The Cobb-Douglas production function is a specific model of production, a model consistent with the neoclassical growth model. Letting Y_t , N_t and K_t denote the period t output, employment, and capital levels, the Cobb-Douglas production relationship can be presented as $Y_t = AN_t^\alpha K_t^\beta$. Taking the natural log and differentiating, one finds α is the

“elasticity of output with respect to labor,” while β is the “elasticity of output with respect to capital.” Production exhibits constant returns to scale if and only if $\alpha + \beta = 1$. Deriving these theoretical results allows intermediate macro and econometrics students to review concepts of elasticity and constant returns to scale, and apply fundamental mathematical tools.

What are the values of α and β for the U.S. economy? Does the U.S. economy exhibit constant returns to scale? By seeking answers to these questions, students are exposed to a variety of interesting empirical and theoretical issues as estimates are sought for α and β . The most direct way of obtaining an econometric model that can be used to estimate the parameters α and β is to take the natural log of the Cobb Douglas production function estimate the model $\ln Y_t = \ln A + \alpha \ln N_t + \beta \ln K_t$. This model implicitly assumes the technology level A is constant, and that technical progress is “Hicks Neutral,” meaning it is not embodied in either labor or capital.⁶ By regressing $\ln Y_t$ on $\ln N_t$ and $\ln K_t$, the ordinary least squares procedure chooses the values of α , β , and A that best fit the data. Doing so, we obtain

$$(5) \quad \ln Y_t = -4.82 + 0.61[\ln N_t] + 0.69[\ln K_t], \quad R^2 = 0.9974.$$

(0.83)*** (0.14)*** (0.08)***

Because the standard errors are small relative to the estimated parameters, the estimates of α and β are statistically significant. The estimate $\alpha = .61$ indicates a 10 percent increase in employment would increase output by 6 percent. This inelastic relationship between labor and output indicates there are decreasing returns to labor. Similarly, the estimate $\beta = .69$ indicates decreasing returns to capital. The sum $\alpha + \beta = 1.30$ indicates increasing returns to scale.

Intriligator (1978, p. 267) notes that the estimation procedure we have just described is questionable because of multiple potential econometric problems. Explanatory variables should be independent of each other, but labor and capital levels are simultaneously chosen by producers seeking profits, so they are not likely to be independent. This “endogeneity problem” exists because output, labor, and capital levels are all determined from the producer optimization problem. Also, the model may be misspecified, especially because the level of technology may not be constant as assumed. Any of these econometric problems can bias the coefficient estimates.

⁶ For a clear discussion of the difference between technical progress that is “Hicks neutral” versus “Harrod neutral” versus “Solow neutral,” see Allen (1969, pp. 236-238).

Economic theory can help us find a different way to estimate the parameters of interest, so as to address the endogeneity econometric problems (Intriligator, 1978, p. 267-68). In particular, the assumption that producers maximize profits suggests particular economic relationships. The goal of obtaining an equation with better econometric properties is an opportunity to review the producer optimization problem with intermediate level students. The profit of the firm can be presented as $\Pi = PY - WN - RK$, where P is the product price level, W is the nominal wage level, and R is the nominal rental rate paid for capital. If production is of the Cobb-Douglas form, then students should be able to show that profit maximization with respect to the labor choice implies $\alpha = WN / PY$, while profit maximization with respect to the capital choice implies $\beta = RK / PY$.

While data on payments to capital is difficult to obtain, we can get an aggregate labor payment from the U.S. Department of Commerce (2005d) called the “compensation of employees,” and we can construct the share of output paid to labor (i.e., WN / PY) by dividing this labor payment by nominal GDP. This labor share is roughly constant. Using the average of this share over the 1948-2005 period for our estimate we obtain the estimate $\alpha = .57$. If we add the assumption of constant returns to scale then we can obtain an estimate for $\beta = RK / PY$ of $\beta = 1 - \alpha = .43$.

With α and β determined by the assumptions of profit maximization and constant returns to scale, we can still further adjust the model by now allowing for the possibility that technology may change over time. Does technology change over time? If it does, is its growth rate constant or does it vary? Using the estimates we have obtained for α and β , we can examine these questions.

We can model technical change using the exponential relationship $A_t = a_0 e^{a_1 t + a_2 t^2 + a_3 t^3}$. The previous descriptive work helps students see that if the level of technology does not change over time, then a_1 , a_2 , and a_3 will be zero. If technology is growing at a constant rate, then a_1 will be positive, but a_2 , and a_3 will be zero. Adding a_2 allows for the possibility that the rate of technological change is either increasing or decreasing over time, while adding a_3 allows the rate of technological change to both increase and decrease.

Using this model for technical change, the production function becomes

$Y_t = a_0 e^{a_1 t + a_2 t^2 + a_3 t^3} N^\alpha K^\beta$. Taking the natural log and then time derivative, we obtain a standard “growth accounting” relationship,

$$\left[\frac{1}{Y_t} \left[\frac{dY_t}{dt} \right] \right] = \left[a_1 + 2a_2 t + 3a_3 t^2 \right] + \alpha \left[\frac{1}{N_t} \left[\frac{dN_t}{dt} \right] \right] + \beta \left[\frac{1}{K_t} \left[\frac{dK_t}{dt} \right] \right].$$

Moving the labor and capital terms to the left side of the equation, substituting our estimates $\alpha = .57$ and $\beta = .43$, and making the substitutions $b_1 = a_1$, $b_2 = 2a_2$, and $b_3 = 3a_3$, we obtain $\left[\frac{1}{Y_t} \left[\frac{dY_t}{dt} \right] \right] - .57 \left[\frac{1}{N_t} \left[\frac{dN_t}{dt} \right] \right] + .43 \left[\frac{1}{K_t} \left[\frac{dK_t}{dt} \right] \right] = b_1 + b_2 t + b_3 t^2$. The right side of this equation is the economy’s rate of technical change, which can change over time. We can estimate this equation by first constructing the variable on the left side of this equation and then regressing it on t and t^2 . Doing so, we find that the coefficient b_3 is not significantly different than zero, and indication that inclusion of the variable t^2 does not help explain the data. Dropping t^2 and rerunning the regression, we also find including t does not help explain the data. To test whether the constant term $b_1 = a_1$ is significant, we can regress the left side variable on a column of 1s in excel. Doing so, we obtain the estimate $b_1 = 0.0099$ with a p-value that indicates this estimate is significantly different than zero. That is, we find that the rate of technical change is constant, growing at about 1 percent per year.

In summary, assuming constant returns to scale and producer profit maximization, we find the Cobb Douglas production function $Y_t = AN^\alpha K^\beta$ results in the following “growth accounting” relationship

$$(6) \quad \frac{1}{Y_t} \frac{dY_t}{dt} = \frac{1}{A_t} \frac{dA_t}{dt} + \alpha \frac{dN_t}{N_t} + \beta \frac{dK_t}{K_t} \Rightarrow 0.0334 = 0.0099 + 0.57(.0170) + 0.43(0.0322)$$

$$\Rightarrow 100.0\% = 29.7\% + 28.9\% + 41.5\%$$

The 3.34 percent average annual real GDP growth rate is decomposed into the portion attributable to technical change, the portion attributable to labor growth, and the portion attributable to capital growth. Capital growth makes the largest contribution, followed technological improvement, then by labor growth.

In contrast to the Hicks Neutral technical change we just estimated, growth models typically assume a constant rate technical change is “Harrod neutral,” or embodied in labor. This is because the Harrod neutral assumption provides a model that roughly captures the path

actually followed by the U.S. economy. When technical change is Harrod neutral, the Cobb Douglas production function is $Y_t = (AN_t)^\alpha K_t^\beta$. The quantity AN_t is called the “effective” labor level. Taking the natural log and differentiating with respect to time, we obtain the model $[1/Y_t][dY_t/dt] = \alpha[1/A_t][dA_t/dt] + \alpha[1/N_t][dN_t/dt] + \beta[1/K_t][dK_t/dt]$.

To estimate this model, we must run the same regression as ran to estimate model with Hicks neutral change. The difference is that the estimate 0.0099 is now equal to $\alpha[1/A_t][dA_t/dt]$. Using the estimate of 0.57 for α , we find that the Harrod neutral rate of technical change is $[1/A_t][dA_t/dt] = 0.0099/.57 = 0.0174$. Adding this 0.0174 rate of technical change with the 0.0170 growth rate of labor, we obtain a 0.0344 estimate of the growth rate of the effective labor force AN . In the theory provided by the standard growth model, effective labor, capital and output each grow at the same constant rate in the model’s steady state. Our empirical work indicates this model fits the data in that the estimated rates are 3.44, 3.22 and 3.34 percent, respectively. The theory also indicates that growth rates for output per unit of labor Y_t/N_t , the real wage paid to labor will each grow at the rate of technical change. Our data indicates these measures of the average standard of living grow at average rates of 1.65 and 1.77 percent, respectively, which are very near our 1.74 percent estimate for the rate of technical change.

To this point, we have estimated paths for the rate of technical change under the assumption of constant returns to scale. But, does the U.S. economy exhibit constant returns to scale? One way to examine this issue is to estimate the model

$$[1/Y_t][dY_t/dt] - .57[1/N_t][dN_t/dt] = [\alpha b_1 + \alpha b_2 t + \alpha b_3 t^2] + \beta[1/K_t][dK_t/dt].$$

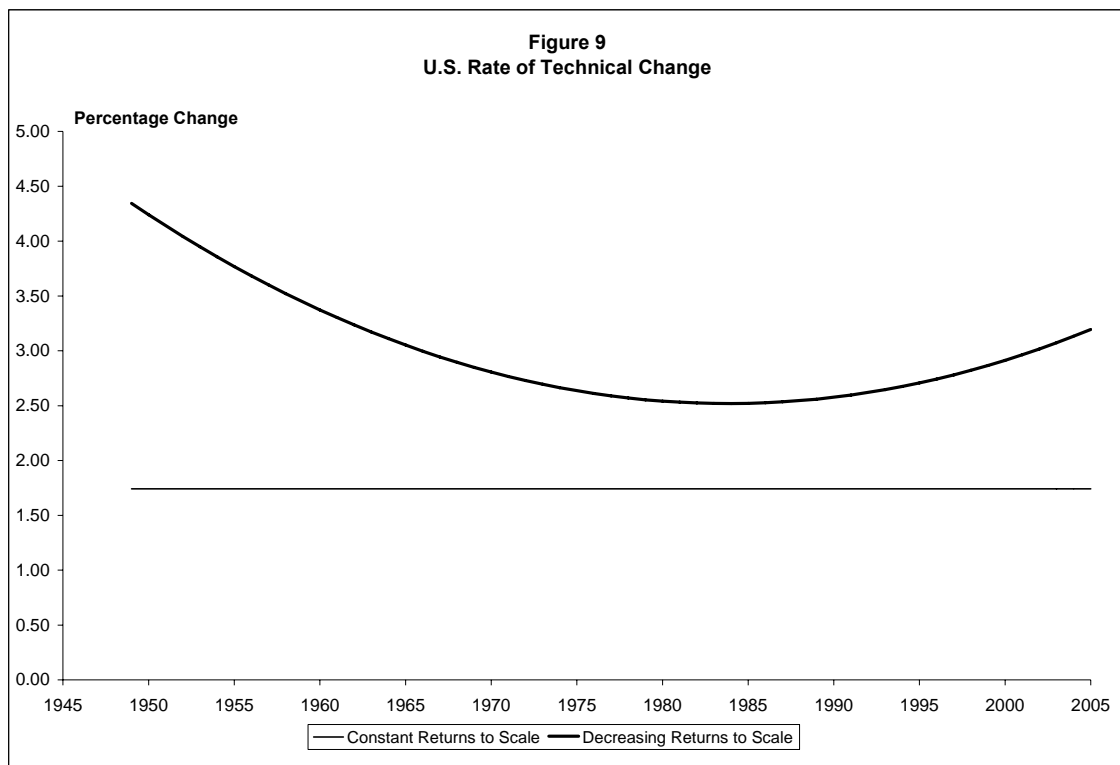
We keep the estimate $\alpha = .57$ obtained from the assumption of profit maximization with respect to labor. This allows us to avoid an endogeneity econometric problem because we can avoid using both the growth rate of labor and growth rate of capital as explanatory variables. By placing the growth rate of capital on the right side of the regression equation, we can estimate the value directly, while also estimating the rate of technical change. Running the regression, we obtain

$$(7) \quad \frac{1}{Y_t} \frac{dY_t}{dt} = [0.025 - 0.0006t + .000009t^2] + .021 \frac{1}{K_t} \frac{dK_t}{dt}, \quad R^2 = 0.6601$$

(.004)*** (.0003)*** (.000005)* (.022)***

The best fit estimate $\beta = .21$ is substantially lower than the $\beta = 1 - \alpha = .43$ previously obtained from the constant returns to scale assumption. Here, $\alpha + \beta = .78$, an indication of decreasing returns to scale. This result provides inspiration to have students use their math skills to show that decreasing returns to scale indicates there are economic profits to be earned, a return that accrues to unrecognized factors of production (e.g. location).

Figure 9 presents the Harrod neutral rate of technical change obtained from the constant returns to scale model, and from the decreasing returns to scale model. Because the reduction in the estimate for β indicates capital growth contributes less significantly to the growth rate of output, the estimated contribution of technical change increases for most time periods. Notice, that the latter indicates the rate of technical change was relatively high after World War II, decreasing to a low in the early 1980s, and then increasing to present. Productivity gains in manufacturing, playing themselves out after World War II, might explain the gradual decline, while productivity gains from the computing power and information technologies might explain the more recent increase in technical change.



4. Concluding Remarks

The Association to Advance Collegiate Schools of Business mandates that “Every school should enunciate and measure its learning goals” (AACSB, 2008, p.60). At the University of Nevada, Reno, one of five learning goals for business school students is, “Students will possess analytical and critical thinking skills.” Most programs in economics and business have a similar desired learning outcome. While there are particular courses that obviously focus on the development of such skills, this learning goal is an example of one that spans the curriculum. Accreditation bodies like the AACSB are increasingly looking for institutions to “close the loop” by using feedback obtained from performance measurement to improve the curriculum. Feedback from performance in senior year capstone courses at the University of Nevada, Reno has suggested that students learning can be enhanced by additional sophomore and junior year applications of analytical skills taught in freshman-sophomore math and statistics courses. Altering a macroeconomics course to include some empirical work, as suggested here, is one way to provide an additional opportunity to develop analytical skills.

Carlson and Watts (1999) find the case study approach useful for teaching statistics, and we have found the empirical examination of growth theory useful for the same reasons. While their case studies focused on markets for various goods, our approach can be considered a case study focusing on the economic growth of the U.S.. They found students were more motivated to learn the tools when the tools were being applied to answer economic questions. We have found the same. They also noted that students obtained a greater appreciation for the economic theory because it impacted the use of the statistical methods, and this is in accord with our experience.

Even before courses in statistics or econometrics, introductory macro students can get some exposure to working with data by describing how an economy has grown, as demonstrated here. In an intermediate macro course, the empirical work provides a welcomed break from analyzing yet another theoretical model. Even students in master degree programs can benefit from this kind of activity in a macroeconomics course, for it is much more likely that they will later use the empirical skills than the macro-modeling skills. Employers like students who can do, and doing something well tends to require much practice. The empirical examination of economic growth is one avenue for providing this practice.

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